MA 141, SECTION 16, FALL 2004, TEST II INSTRUCTOR: JAMES COO[illegible]

Work each problem clearly and seperately on your own paper. If you use a pen please work on only one side of the paper.

1 | 7pts | Use the squeeze theorem to prove $\lim_{x\to\infty}\left(\frac{\sin(x)}{x^2}\right) = 0$.

2 | 8pts. | Answer each question in <u>words</u> complemented with a <u>graph.</u>

i) What is the definition of a continous function?
ii) What is the definition of the derivative of $f(x)$ at $x=a$?
iii) How does the derivative of a function relate to the tangent line at $(a, f(a))$ of the graph $y=f(x)$?
iv.) What is an example of a function which is continous but not differentiable? Recall that differentiable means you can differentiate the function on its doma[illegible]

3 | 6pts. | Prove from the definition of the derivative that $\frac{d}{dx}(e^x) = e^x$. Recall that we defined e to be the number such that the following limit is true; $\lim_{h\to 0}\left(\frac{e^h-1}{h}\right) = 1$.

4 | 16pts | Let $s(t) = t^2-2t+1$ be the position of a particle at time t. find the velocity and acceleration at time t. Then find the tangent to the curve $y=s(t)$ in the $(t\,y)$-plane at the point $(1,4)$. Graph the curve and this tangent line.

5 | 20pts | Derive the following:

a.) $\frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$	c.) $\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)$
b) $\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$	d.) $\frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$

6 | 40pts. | Calculate the derivatives below using whatever methods appropriate notice you can use problem 5 to simplify several of these.

a.) $\frac{d}{db}\left(cb^2+2xd\right)$	e.) $\frac{d}{dx}\left(x\sin(x)\cos(x)\right)$	i) $\frac{d}{du}\left(\sin(u)\csc(u)\right)$
b.) $\frac{d}{dx}\left(\sqrt[3]{x^2}\right)$	f.) $\frac{d}{dx}\left(\frac{x+3}{\cos(x)}\right)$	j) $\frac{d}{dx}\left(\frac{\sin(x)}{\tan(x)+e^x}\right)$
c.) $\frac{d}{ds}\left(\frac{1}{s^2}\right)$	g.) $\frac{d}{dx}\left(ae^x(x^2+3)\right)$	
d.) $\frac{d}{dt}\left(e^t\sec(t)\right)$	h.) $\frac{d}{dx}\left(\frac{x+x^2}{\sqrt{x}}\right)$	

<u>Extra Credit's On the Chalk Board</u>