MATH 121	FEB. 13-15, 2010	QUIZ 2

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks and enjoy. This quiz is due by the start of class Monday Feb. 15, 2010. If you cannot make it to lecture then you may also turn this quiz in at my office before lecture (just leave it in one of the boxes).

Problem 1 [30pts] Factor the polynomials below as much as is possible over $\mathbb{R}$.

(a.) $f_1(x) = x^2 + 5x + 6$

$= (x+3)(x+2)$

(b.) $f_2(x) = x^4 - 4$

$= (x-2)(x+2)$

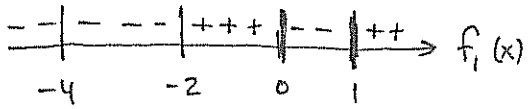
(c.) $f_3(x) = (x+2)x^2 - (x+2)$

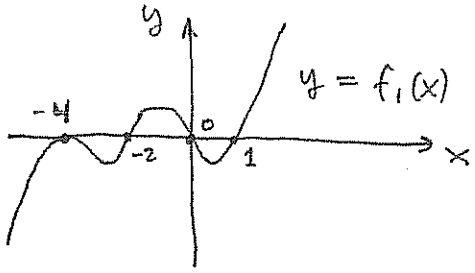
$= (x+2)(x^2 - 1)$

$= (x+2)(x+1)(x-1)$

Problem 2 [40pts] For each of the functions given below draw the sign chart and then use the sign chart to help sketch the graph of the given function.

(a.) $f_1(x) = (x+4)^4(x+2)^3x(x-1)$

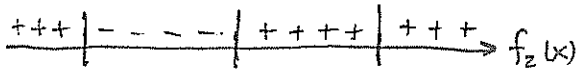


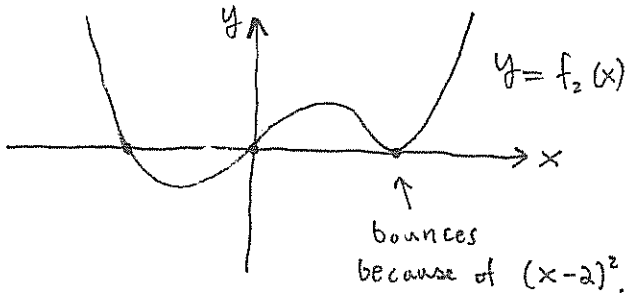


(b.) $f_2(x) = x(x-2)^2(x+2)^3(x^2+4x+5) = x(x-2)^2(x+2)^3\left((x+2)^2+1\right)$

irreducible!

$f_2(-3) = (-3)(-5)^2(-1)^3(2) > 0$





(c.) $f_3(x) = \frac{x(x^2+1)}{x-2}$

not on test.

do not grade.

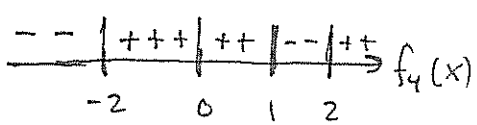
(do not grade)

not on test 1, Spring 2010.

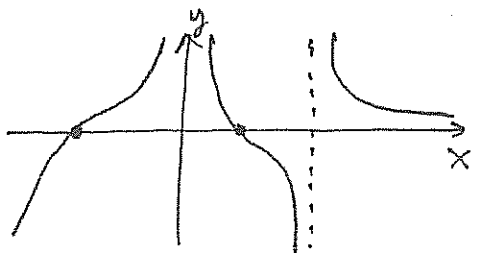
(d.) $f_4(x) = \frac{1}{x-2} + \frac{1}{x^2} = \frac{x^2+x-2}{x^2(x-2)} = \frac{(x+2)(x-1)}{x^2(x-2)}$

Zeros at $x = -2$ and 1

VA's at $x = 0$ and $x = 2$.



$f_4(-3) = \frac{-1}{5} + \frac{1}{9} < 0$



Problem 3 [10pts] Let $g(x) = x^4 + 3x^3 + 8x^2 + 12x + 16$. Note that $g(2i) = 0$ where $i^2 = -1$. Factor $g(x)$ by using a theorem about complex roots of real polynomials.

$g(2i) = 0 \Rightarrow g(-2i) = 0 \Rightarrow (x-2i)(x+2i) = \underbrace{x^2+4}$ factor of $g(x)$.

$$
\begin{array}{r}
x^2+3x+4 \\
x^2+4 \;\overline{)\; x^4+3x^3+8x^2+12x+16} \\
\underline{x^4+4x^2} \\
3x^3+4x^2+12x+16 \\
\underline{3x^3+12x} \\
4x^2+16 \\
\underline{4x^2+16} \\
0
\end{array}
$$

← good check on my calculations.

grader: only take off 1 pt.

$\therefore\ g(x) = (x^2+4)(x^2+3x+4)$

$= \boxed{(x^2+4)(x+3)(x+1)}$

Problem 4 [20pts] Write down an example of a polynomial or rational function as instructed below:

(a.) polynomial function $p(x)$ with zeros at $x = 1, 2, 7$.

$\boxed{P(x) = (x-1)(x-2)(x-7)}$

(b.) polynomial function $q(x)$ with a complex zero of $x = i$ and a real zero of $x = 3$ such that the graph "bounces" at $(3, 0)$

$\boxed{q(x) = (x^2+1)(x-3)^2}$

→ x^2+1 in factorization.

(c.) rational function $f(x)$ with zero $x = 2$ and vertical asymptotes $x = 3$ and $x = -1$

$$\boxed{f(x) = \frac{x-2}{(x-3)(x+1)}}$$

(d.) rational function $g(x)$ with no zeros and a single vertical asymptotes at $x = 3$ such that ~~graph(g)~~ $= (0, \infty)$.

range$(g) = (0, \infty)$ oops!

(Don't grade part d.)