No graphing calculators or electronic communication of any kind. Credit will be awarded for correct content and clarity of presentation. This quiz has 100 required points and 10 bonus. Try to at least attempt each part. Please clearly box or circle your answer in each question.

1.[5pts] [section 2.2] Find the domain for the function.

$$f(x) = \sqrt{x-4} \qquad \text{need } x - 4 \geq 0$$

$$\therefore \boxed{\text{dom}(f) = [4, \infty)}$$

2.[5pts] [section 2.3] State the domain and range of the graphed function.
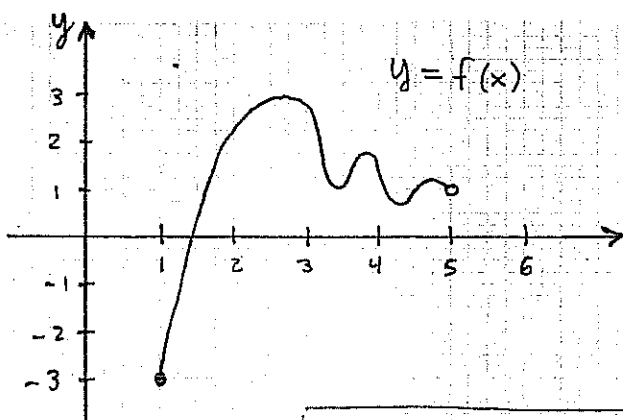
$$\text{inputs} \longrightarrow \boxed{\text{dom}(f) = [1, 5)}$$
$$\text{outputs} \longrightarrow \boxed{\text{range}(f) = [-3, 3]}$$

3. [10pts] Find the equation for the line passing through (1,0) and (2,3). Please leave your answer in slope-intercept form.

$$y = mx + b$$
$$(1,0) \Rightarrow 0 = m + b \Rightarrow \underline{m = -b}.$$
$$(2,3) \Rightarrow 3 = 2m + b \Rightarrow 3 = -2b + b = -b$$
$$\text{Thus} \quad b = -3 \quad \text{and} \quad m = 3$$

$$\therefore \boxed{y = 3x - 3}$$

4.[5pts] [section 3.2] Write a formula for a polynomial function $f(x)$ having zeros at $x = 1, 2, 3$.

$$\underline{f(x) = (x-1)(x-2)(x-3)}.$$

$$(\text{using Factor Th}^m)$$

5.[10pts] [section 3.1] Write the equation of a quadratic function whose graph has vertex $(-2, 3)$ and passes through the point $(0, 13)$. Please put the answer in standard form.

vertex at $(-2, 3)$ gives

$$y = A(x+2)^2 + 3$$
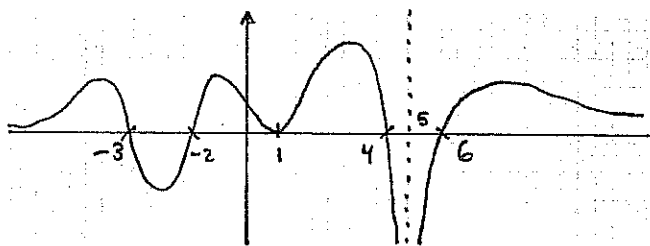
we need to find A. Use the given point $(0, 13)$

$$13 = A(2)^2 + 3$$
$$\Rightarrow 10 = 4A$$
$$\Rightarrow A = {}^{10}/_4 = {}^5/_2.$$

$$\therefore \boxed{y = \frac{5}{2}(x+2)^2 + 3}$$

6. [10pts][section 3.2] Given the graph below circle the function which has a matching graph.

$$f_1(x) = (x+3)(x+2)(x-1)(x-5)(x-6)(x-4)$$
$$f_2(x) = (x-3)(x-2)(x+1)(x+5)(x+6)(x+4)$$
$$\boxed{f_3(x) = \frac{(x+3)(x+2)(x-1)^2(x-4)(x-6)}{(x-5)^2}}$$

7.[5pts][section 3.3] Does the function $f(x) = 33x^{13} - 20x^2 - 11x - 2$ have $(x-1)$ as a factor? Justify your answer either by a Theorem or by an explicit algebraic calculation.

$$f(1) = 33 - 20 - 11 - 2 = 0$$

$$\therefore (x-1) \text{ factors } f(x).$$

$$(\text{By Factor Th}^m)$$

8.[10pts] Completely factor (over $\mathbb{R}$) the following polynomials:

(a.) $x^2 - 10x + 9 = \boxed{(x-1)(x-9)}$

(b.) $x^3 + x = \boxed{x(x^2+1)}$

9.[10pts][section 3.4] Completely factor $f(x) = x^4 + 5x^3 + 7x^2 + 5x + 6$ given that $f(x)$ has a zero at $x = i = \sqrt{-1}$.

$f(i) = 0 \implies f(-i) = 0$

thus $(x+i)(x-i) = x^2 + 1$ factors $f(x)$.

$$
\begin{array}{r}
x^2 + 5x + 6 \\
x^2 + 1 \overline{\smash{)}\ x^4 + 5x^3 + 7x^2 + 5x + 6} \\
\underline{x^4 + x^2} \\
5x^3 + 6x^2 + 5x + 6 \\
\underline{5x^3 + 5x} \\
6x^2 + 6 \\
\underline{6x^2 + 6} \\
\boxed{0} \quad \ddot{\smile}
\end{array}
$$

Thus,
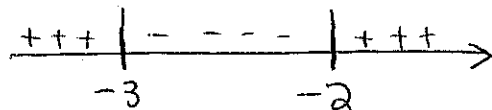$f(x) = (x^2+1)(x^2+5x+6)$
$= \boxed{(x^2+1)(x+3)(x+2)}$

10. [10pts] Solve the following inequality.

$-3x + 2 \le 7 + x$

$-5 \le 4x$
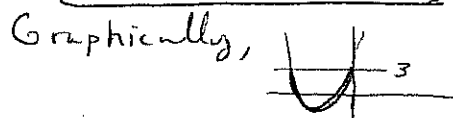
$\boxed{\dfrac{-5}{4} \le x}$

a.k.a.

$\boxed{x \ge -5/4}$

a.k.a $x \in [-5/4, \infty)$.

11. [10pts] Solve the following inequality. Justify your answer either with a sign chart as I used in lecture, or with a table as shown in the textbook. Remember to neatly box your answer.

$x^2 + 5x + 9 \le 3 \iff x^2 + 5x + 6 \le 0$
$\iff (x+3)(x+2) \le 0$

Thus

$\boxed{-3 \le x \le -2}$

Graphically,

12. [10pts] What explicit numerical values for A, B that make the equality below true ?

$\dfrac{x\sqrt{xy}}{y} = x^A y^B$

$x\sqrt{x}\sqrt{y}\ y^{-1} = x^{3/2}\ y^{-1/2} = x^A y^B$

$\boxed{A = 3/2} \quad \boxed{B = -1/2}$

13. [5pts] Give a formula for a rational function which has zeros at 3 and 7, a vertical asymptote at x=4 and a hole in the graph at (2,0). Also, choose your example so that the range of the function includes no negative numbers.

$f(x) = \dfrac{(x-3)^2(x-7)^2(x-2)}{(x-4)^2(x-2)}$

other answers possible but $(x-3)^m, (x-7)^p, (x-4)^k$ must have even m, p, k.

$\dfrac{(x-2)}{(x-2)}$ could be ~~replaced~~ replaced with $\left(\dfrac{x-2}{x-2}\right)^j$ for ~~any~~ $j = 1, 2, \ldots$ etc.