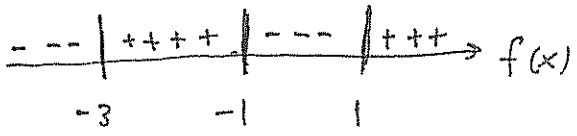
MATH 121	MAR. 3, 2010	QUIZ 4

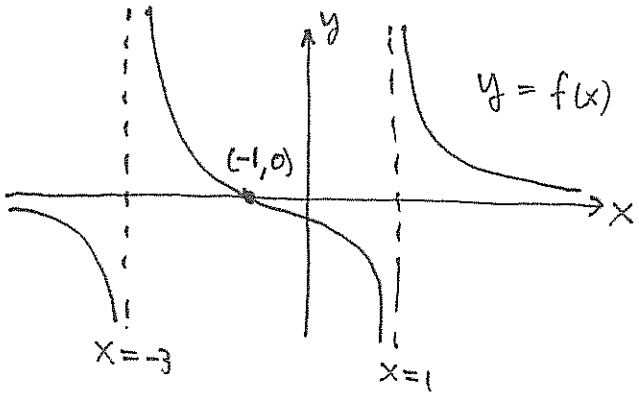
Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks and enjoy.

Problem 1 [50pts] Let $f(x) = \frac{1}{x+3} + \frac{1}{x-1}$. Find all zeros and vertical asymptotes for $f(x)$. Draw the sign-chart and sketch the graph with the vertical asymptotes and zeros clearly labeled. State the domain and range of the function.

$$f(x) = \frac{1}{x+3} + \frac{1}{x-1} = \frac{x-1+x+3}{(x+3)(x-1)} = \frac{2x+2}{(x+3)(x-1)} = \frac{2(x+1)}{(x+3)(x-1)}$$

Thus $f(-1) = 0$ whereas $x = -3$, $x = 1$ give vertical asymptotes.





$\text{dom}(f) = \{x \mid x \neq 1 \text{ and } x \neq -3\}$

$\text{range}(f) = (-\infty, \infty)$.

Problem 2 [10pts] Simplify the rational expression below:

$$\frac{2x^2+2x}{5x+5} = \frac{2(x^2+x)}{5(x+1)} = \frac{2x(x+1)}{5(x+1)} = \boxed{\frac{2x}{5} \text{ or } \frac{2}{5}x}$$

Problem 3 [40pts] Solve the following equations. (find all real solutions)

(a.)

$\frac{1}{x}+3=0$ $\longrightarrow$ $\frac{1}{x} = -3$ $\longrightarrow$ $\boxed{x = \frac{-1}{3}}$

(b.)

$\frac{1}{x}+\frac{1}{x-2}=0$ $\longrightarrow$ $\frac{1}{x} = \frac{-1}{x-2}$

$\longrightarrow$ $x = -(x-2)$ if $x \neq 0$ and $x \neq 2$.

$\longrightarrow$ $2x = 2$

$\longrightarrow$ $\boxed{x = 1}$

(c.)

$\frac{4}{x}-\frac{5}{3}=\frac{x}{6}$ $\longrightarrow$ $\frac{12-5x}{3x} = \frac{x}{6}$

$\longrightarrow$ $72 - 30x = 3x^2$

$\longrightarrow$ $x^2 + 10x - 24 = 0$

$\longrightarrow$ $(x+12)(x-2) = 0$ $\therefore$ $\boxed{x = 2 \text{ or } x = -12}$

Problem 4 [20pts] Write down an example of a rational function $g(x)$ with zeros at $x = 3$ and $x = 4$ and vertical asymptotes at $x = -8$ and $x = 0$ (there are many correct answers).

$g(x) = \frac{(x-3)(x-4)}{(x+8)(x-0)}$ or $\frac{(x-3)^a(x-4)^b}{x^c(x+8)^d}$

for $a, b, c, d = 1, 2, 3, \ldots$

all good answers.