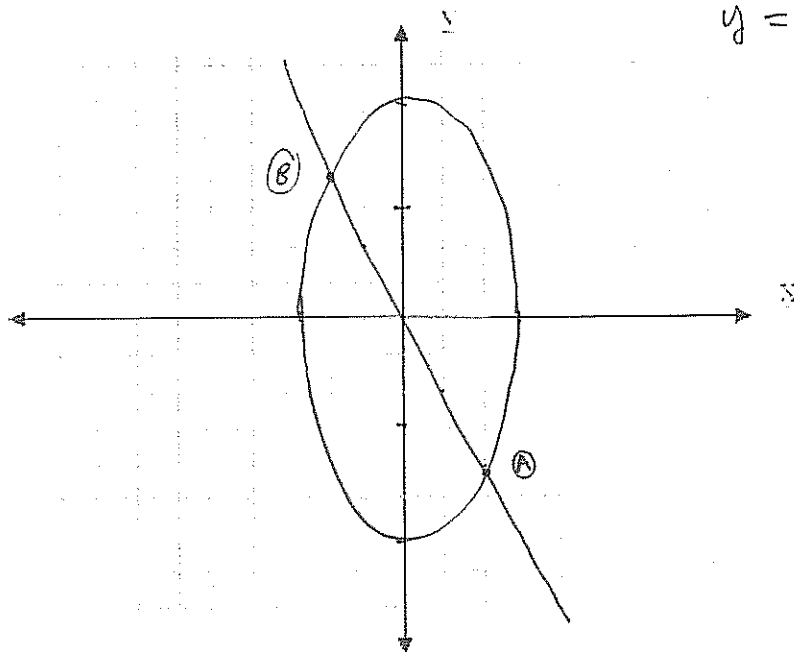Name:

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit.
Thanks and enjoy. Remember, no graphing calculators permitted.

Problem 1 [50pts] Graph $x^2 + y^2/4 = 1$ and $2x + y = 0$. Find the points of intersection algebraically and verify your answer on the graph.

$$y = -2x$$
$$\Downarrow$$
$$x^2 + y^2/4 = 1$$
$$x^2 + \frac{(-2x)^2}{4} = 1$$
$$x^2 + x^2 = 1$$
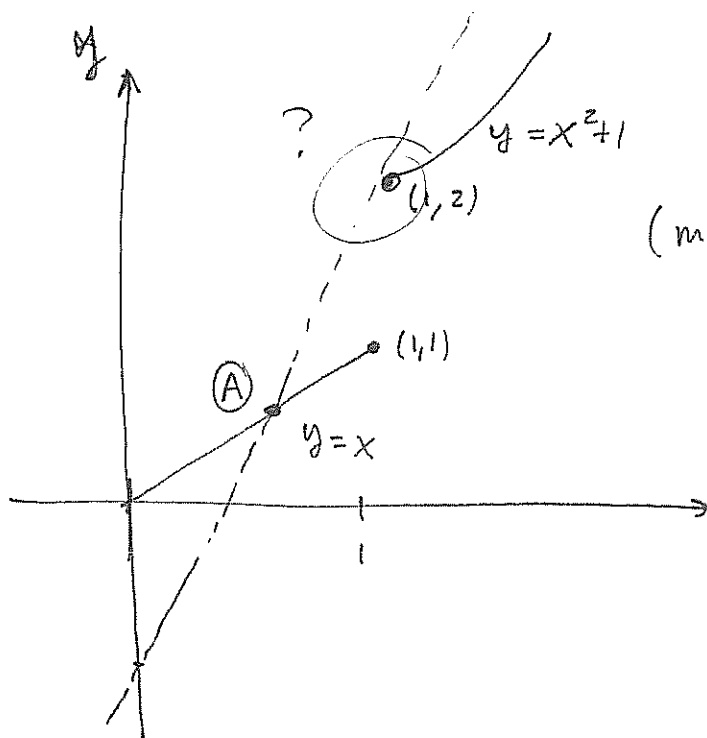$$2x^2 = 1$$
$$x^2 = 1/2$$
$$x = \pm \, 1/\sqrt{2}$$
$$y = \pm \frac{2}{\sqrt{2}}$$

$$\therefore \quad \left(1/\sqrt{2}, \ -2/\sqrt{2} \right) \quad or \quad \left(\frac{-1}{\sqrt{2}}, \frac{2}{\sqrt{2}} \right)$$

Ⓐ   Ⓑ

Problem 2 [50pts] Find the domain and range of $f(x) = \begin{cases} x & 0 \le x \le 1 \\ x^2 + 1 & x > 1 \end{cases}$. Calculate $f(0)$ and $f(3)$. If $g(x) = 3x - 1$ then what are the points of intersection for $y = f(x)$ and $y = g(x)$?

$$f(0) = 0. \quad and \quad f(3) = 3^2 + 1 = 10.$$

y

?

$(1,2)$

$y = x^2 + 1$

(my initial
picture not
right, after
algebra below
it's clear.

Ⓐ

$(1,1)$

$y = x$

1

Let $0 \le x \le 1$, $f(x) = g(x)$ gives

$$x = 3x - 1$$

$$2x = 1 \quad \therefore \quad x = \frac{1}{2} \quad \therefore \quad \boxed{(\frac{1}{2}, \frac{1}{2})} \quad Ⓐ$$

Whereas for $x > 1$, $f(x) = g(x)$ gives

$$x^2 + 1 = 3x - 1$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$\underbrace{x = 1}_{\substack{\text{not in} \\ \text{interval with} \\ x > 1.}} \quad \text{or} \quad \underbrace{x = 2}_{\text{ok.}} \qquad \boxed{(2, 5)} \quad Ⓑ.$$

y

$y = 3x - 1$

$(2,5)$

$(\frac{1}{2}, \frac{1}{2})$

x
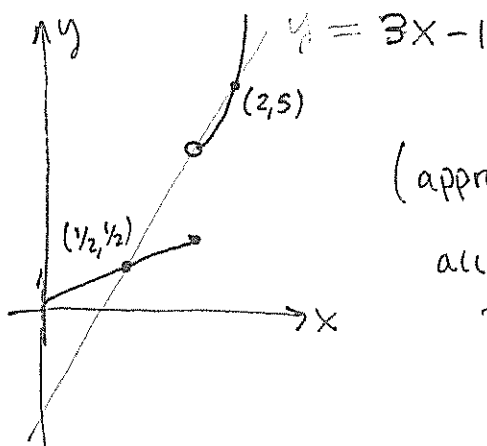
(approximately
accurate
picture)