College Algebra: Quiz

Work these out on separate paper and use ruler to help graph neatly. You may work in groups & with tutors. I need this by Friday 5pm. Thanks!

1.) Solve the inequality below:

$$\frac{(x+3)(x^2-4)}{(x^2+1)(x-6)} \leq 0.$$

2.) Find all sol^n's to $x^3 + 4x^2 + 3x = 0$.

3.) Graph $y = \frac{x}{x^2-9}$ by finding all zeros, vertical asymptotes and drawing the sign-chart to guide your drawing.

4.) Let $f(x) = \sqrt{x}$. Find the eq^n of a graph which is obtained from taking the graph $y = \sqrt{x}$ and shifting it horizontally 3 units right then reflecting that graph about the y-axis.

5.) Let $f(x) = \frac{2+x}{x-1}$. Find the inverse function $f^{-1}(x)$.

6.) Graph $y = 2^x$ and $y = \log_2(x)$ for $0 < x < 4$.

7.) Solve $8^x = 2^{3-x}$ for x.

8.) Simplify $\ln(x^2+1) + \frac{1}{2}\ln(x^2+4x+5) - \ln(x)$ by rewriting these three summands as a single logarithm.

9.) Solve $\ln(x+1) + \ln(x+3) = \ln(3)$.

10.) Solve $e^{2x} - e^x - 6 = 0$. $\left(\text{Hint, substitute } y = e^x \text{ and solve for } y \text{ to start}\right)$

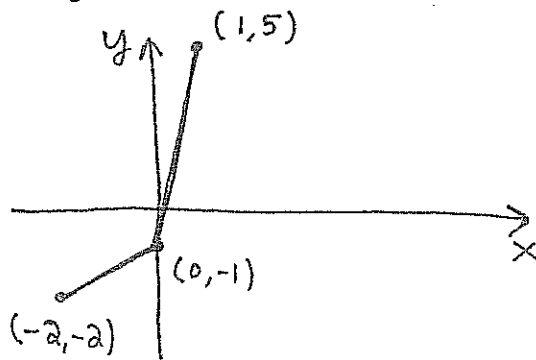11.) Let $f(x) = e^x$ and $g(x) = \sqrt{x}$. Find formulas for
   a.) $(f+g)(x)$
   b.) $(fg)(x)$
   c.) $(f \circ g)(x)$
   d.) $(g \circ f)(x)$

12.) Let $f(x) = 3e^x - 7$. Find the inverse function $f^{-1}(x)$.

13.) Your given the graph of $y = f(x)$ below. Construct
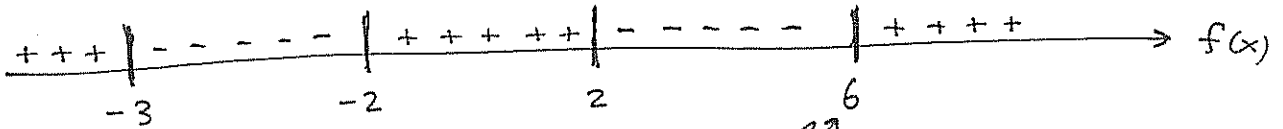the inverse function's graph as best you can from
the given data.

14.) Solve $3^{x^2} = 81$.

15.) Solve $\ln(x+1) = \log_3(x+1)$.

SOLUTION TO THE QUIZ GIVEN IN WEEK BEFORE BREAK:

PROBLEM 1 Solve $\dfrac{(x+3)(x^2-4)}{(x^2+1)(x-6)} \leq 0$.

Sol^n: draw sign-chart after properly factoring the given

expression; $\dfrac{(x+3)(x^2-4)}{(x^2+1)(x-6)} = \dfrac{(x+3)(x+2)(x-2)}{(x^2+1)(x-6)} = f(x)$

$$+++\ \big|\ -----\ \big|\ ++++++\ \big|\ -----\ \big|\ ++++\ \longrightarrow\ f(x)$$
$$\quad\ -3 \qquad\qquad -2 \qquad\qquad 2 \qquad\qquad 6$$

Hence the sol^n is

$$\boxed{[-3,-2]\cup[2,6)}$$

V.A. so it's not included.
$x = -3, -2, 2$ all zeros so they are included since we were given $\leq$ type question.

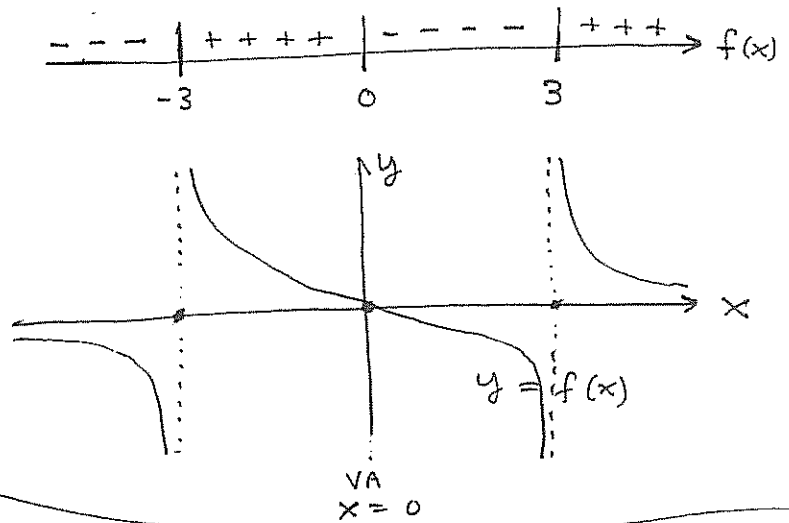**PROBLEM 2** Solve $x^3 + 4x^2 + 3x = 0$

$x^3 + 4x^2 + 3x = x(x^2 + 4x + 3)$
$\qquad\qquad\qquad = x(x+3)(x+1) = 0$
$\qquad\qquad \Rightarrow \boxed{x = 0, -3, -1}$

PROBLEM 3 Graph $y = \dfrac{x}{x^2-9}$

Note $f(x) = \dfrac{x}{x^2-9} = \dfrac{x}{(x+3)(x-3)}$ $\Rightarrow$ VA's at $x = \pm 3$.
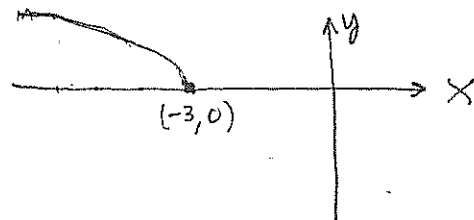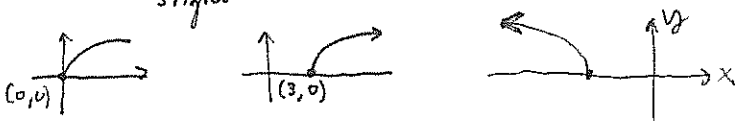ZERO at $x = 0$.

Observe $f(-1) = \dfrac{-1}{-8} > 0$
and $f(1) = \dfrac{1}{-8} < 0$
and $f(-4) = \dfrac{-4}{16-9} < 0$
whereas $f(4) = \dfrac{4}{16-9} > 0$

$$----\ \big|\ ++++\ \big|\ ----\ \big|\ +++\ \longrightarrow\ f(x)$$
$$\qquad -3 \qquad\quad 0 \qquad\quad 3$$

$y = f(x)$
VA $x = 0$

PROBLEM 4 Graph the eqn $y = \sqrt{x}$ shifted 3 units right and then reflected across y-axis

$\sqrt{x} \xrightarrow[\text{3 right}]{\text{shift}} \sqrt{x-3} \xrightarrow{\text{reflect}} \sqrt{-(x-3)}$

$(0,0)$ $\qquad$ $(3,0)$

$(-3,0)$

PROBLEM 5 | Find inverse function of $f(x) = \dfrac{2+x}{x-1}$

Let $y = \dfrac{2+x}{x-1}$ swith $x \longleftrightarrow y \longrightarrow x = \dfrac{2+y}{y-1}$
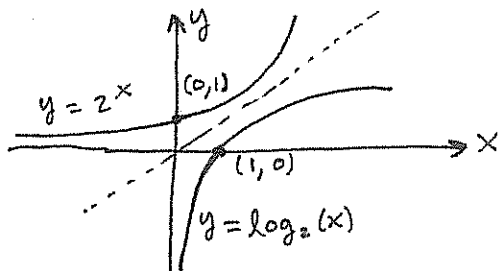
$$\Rightarrow x(y-1) = 2+y$$
$$\Rightarrow yx - y = x+2$$
$$\Rightarrow y(x-1) = x+2$$
$$\Rightarrow y = \dfrac{x+2}{x-1} \qquad \therefore \quad \boxed{f^{-1}(x) = \dfrac{x+2}{x-1}}$$

PROBLEM 6 | Graph $y = 2^x$ and $y = \log_2(x)$ for $0 < x < 4$

$y = 2^x$ $(0,1)$

$(1, 0)$

$y = \log_2(x)$

PROBLEM 7 | Solve $8^x = 2^{3-x}$ for x

Note $8^x = 2^{3-x} \Rightarrow \ln(8^x) = \ln(2^{3-x})$
$$\Rightarrow x \ln(8) = (3-x)\ln(2)$$
$$\Rightarrow x(\ln 8) + x\ln(2) = 3\ln(2)$$
$$\Rightarrow x(\ln(8) + \ln(2)) = 3\ln(2)$$
$$\Rightarrow x = \dfrac{3\ln(2)}{4\ln(2)}$$
$$\therefore \quad \boxed{x = 3/4}$$

note $\ln(8) = \ln(2^3)$
$\qquad = 3\ln(2)$.

Alternatively,
$$8^x = 2^{3-x} \Rightarrow (2^3)^x = 2^{3-x}$$
$$\Rightarrow 2^{3x} = 2^{3-x}$$
$$\Rightarrow 3x = 3-x \qquad \Big\} \quad \text{1-1 property for exponential fnct.}$$
$$\Rightarrow 4x = 3$$
$$\Rightarrow \underline{x = 3/4}.$$

PROBLEM 8) Simplify $\ln(x^2+1) + \frac{1}{2}\ln(x^2+4x+5) - \ln(x) = y$

$$y = \ln(x^2+1) + \ln\sqrt{x^2+4x+5} - \ln(x)$$

$$y = \ln\left[\frac{(x^2+1)\sqrt{x^2+4x+5}}{x}\right]$$

PROBLEM 9) Solve $\ln(x+1) + \ln(x+3) = \ln(3)$

$$\ln\big((x+1)(x+3)\big) = \ln(3)$$

$$\Rightarrow (x+1)(x+3) = 3$$

$$\Rightarrow x^2 + 4x + 3 = 3$$

$$\Rightarrow x(x+4) = 0$$

$$\Rightarrow \underline{x=0} \quad \text{or} \quad \underline{x=-4}$$

1-1 property of natural logarithm.

Note, $\ln(1) + \ln(3) \neq \ln(3)$ however, $\ln(-4+1) + \ln(-4+3)$ d.n.e.

not real #'s. throw out $x=-4$ sol^n.

$$\therefore \boxed{x = 0}$$

PROBLEM 10) Solve $e^{2x} - e^x - 6 = 0$

Let $y = e^x$ then $y^2 = e^x e^x = e^{2x}$ hence $y^2 - y - 6 = 0$

factoring yields $(y-3)(y+2) = 0$ $\therefore$ $y = 3$ or $y = -2$

but, $y = e^x$ and $e^x > 0$ so $y = 3$ is only

interesting sol^n : $3 = e^x \Rightarrow \boxed{x = \ln(3)}$

PROBLEM 11) Let $f(x) = e^x$ and $g(x) = \sqrt{x}$ calculate formulas for $f+g$, fg, $f\circ g$ and $g\circ f$

$$(f+g)(x) = e^x + \sqrt{x}.$$

$$(fg)(x) = e^x\sqrt{x}.$$

$$(f\circ g)(x) = f(g(x)) = f(\sqrt{x}) = e^{\sqrt{x}}.$$

$$(g\circ f)(x) = g(f(x)) = g(e^x) = \sqrt{e^x} = (e^x)^{1/2} = e^{x/2}$$

all fine answers.

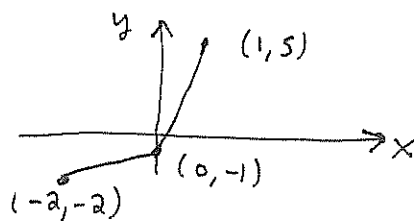PROBLEM 12) Find inverse fnct. of $f(x) = 3e^x - 7$

Let $y = 3e^x - 7$. Switch $x \longleftrightarrow y$, $x = 3e^y - 7$

Solve for y,

$$3e^y = x + 7 \implies e^y = \frac{x+7}{3} \implies y = \ln\left(\frac{x+7}{3}\right)$$
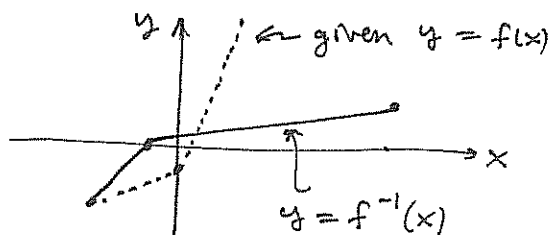
$$\therefore \boxed{f^{-1}(x) = \ln\left(\frac{x+7}{3}\right)}$$

PROBLEM 13) Given graph of $y = f(x)$ construct graph $y = f^{-1}(x)$

Note $(-2,-2), (1,5), (0,-1) \in$ graph (f)

$\implies (-2,-2), (5,1), (-1,0) \in$ graph (f^{-1})

given $y = f(x)$

$y = f^{-1}(x)$

PROBLEM 14) Solve $3^{x^2} = 81$

Note $81 = 9 \cdot 9 = 3^2 \cdot 3^2 = 3^4$ thus $3^{x^2} = 3^4 \implies x^2 = 4$

$$\implies \boxed{x = \pm 2}$$

PROBLEM 15) Solve $\ln(x+1) = \log_3(x+1)$

Note $\ln(x+1) = \log_3(x+1) = \dfrac{\ln(x+1)}{\ln(3)}$ $\longleftarrow$ change of base formula.

$$\implies \underbrace{\ln(x+1)}_{\substack{\text{must} \\ \text{be} \\ \text{zero.}}} \underbrace{\left[1 - \frac{1}{\ln(3)}\right]}_{\text{non zero.}} = 0$$

$$\ln(x+1) = 0 \implies e^{\ln(x+1)} = e^0 \implies x+1 = 0 \therefore \boxed{x = -1}$$