§1.7 #10 p. 81	Given the parametrized curve $x = 4\cos\theta$ and $y = 5\sin\theta$ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ find the Cartesian eq^{n} of curve and sketch it.

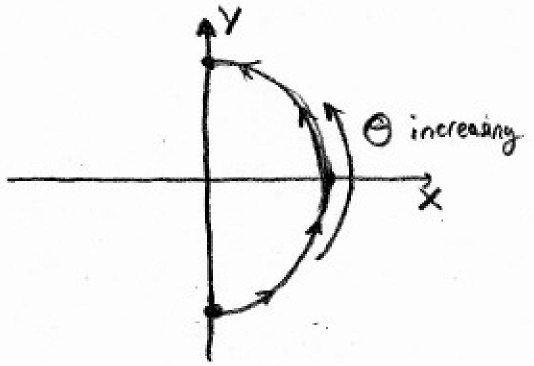
Cartesian coordinates are x and y, the cartesian eq^{n} of a curve only involves x & y. We need to eliminate θ. Lets observe,

$$\cos\theta = \frac{x}{4} \quad \& \quad \sin\theta = \frac{y}{5}$$

And we know $\sin^2\theta + \cos^2\theta = 1$ correct? So that allows us to eliminate θ,

$$\sin^2\theta + \cos^2\theta = \left(\frac{y}{5}\right)^2 + \left(\frac{x}{4}\right)^2 = 1$$

This is an ellipse; $\boxed{\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1}$ (actually $\frac{1}{2}$ an ellipse $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$.)



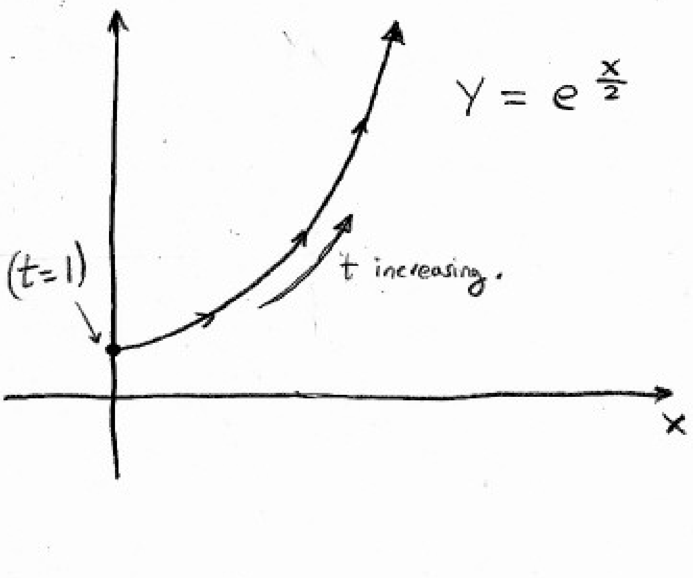
θ	x	y
$-\frac{\pi}{2}$	0	-5
0	4	0
$\frac{\pi}{2}$	0	5

§1.7 #12 p. 81	$x = \ln(t)$ $y = \sqrt{t}$	for $t \ge 1$	Find Cartesian form of eq^{n} and sketch graph

We need to eliminate t somehow. One way is to solve x for t and substitute that into y.

$$x = \ln(t) \implies e^x = e^{\ln(t)} = t$$

$$\therefore \; y = \sqrt{t} = \sqrt{e^x} = (e^x)^{\frac{1}{2}} = \boxed{e^{\frac{x}{2}} = y}$$



t	x	y
1	0	1
2	ln(2)	$\sqrt{2}$
4	ln(4)	2
e^2	2	e

§1.7 #30c p.82

A projectile is fired at angle α above the horizontal with a velocity V. Its equations of motion are

$$x = (V_0 \cos\alpha)t \qquad Y = (V_0 \sin\alpha)t - \tfrac{1}{2} g t^2 \qquad g = 9.8 \tfrac{m}{s^2}$$

Notice that time t is the parameter. Show that this motion follows some parabola by eliminating t to give the Cartesian form of the eq^{n}. (You can do a) & b.) in your physics course ü)

Solve x for t,

$$t = \frac{x}{V_0 \cos\alpha}$$

Substitute into $Y = (V_0 \sin\alpha)t - \frac{1}{2} g t^2$; this eliminates t,

$$Y = (V_0 \sin\alpha)\frac{x}{V_0 \cos\alpha} - \frac{1}{2} g \left(\frac{x}{V_0 \cos\alpha}\right)^2$$

$$\boxed{Y = (\tan\alpha)\, x - \frac{g}{2V_0^2 \cos^2\alpha} x^2}$$

this is a quadratic which has a parabola for its graph (assuming $\alpha \neq 0$)

BONUS PROJECT: Work out neatly the "Taylor Polynomials" Laboratory Project on p. 257-258. (worth 3-times the usual credit)

§3.8 #2 p. 256 Let $f(x) = \ln(x)$ find its linearization at $x=1$, I call this $L_f^1(x)$ in notes.

$$f'(x) = \frac{1}{x} \quad \Rightarrow \quad f'(1) = \frac{1}{1} = 1$$

Thus the linearization of $\ln(x)$ at $x=1$ is,

$$f(x) \cong f(1) + f'(1)(x-1)$$

tangent line to $y=f(x)$ at $(1, f(1))$.

$$= \ln(1) + 1(x-1)$$

$$= \boxed{x - 1 = L_f^1(x) \cong \ln(x)}$$

This is the best linear approximation of natural log at $x=1$. As we get away from $x=1$ this will become less & less accurate.

§3.8 #4 p. 256 Let $f(x) = \sqrt[3]{x}$ find the best linear approx. of f at $x=-8$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3}\left(\frac{1}{\sqrt[3]{x^2}}\right)$$

$$f'(-8) = \frac{1}{3}\left(\frac{1}{\sqrt[3]{(-8)^2}}\right) = \frac{1}{3}\left(\frac{1}{\sqrt[3]{64}}\right) = \frac{1}{3}\frac{1}{4} = \frac{1}{12}$$

Thus we find,

$$f(x) \cong f(-8) + f'(-8)(x+8)$$

$$= \sqrt[3]{-8} + \frac{1}{12}(x+8)$$

$$= \boxed{-2 + \frac{1}{12}(x+8) \cong \sqrt[3]{x}}$$

- If you can find the tangent line then you can find the linearization of a function at some point. Nothing new, except the realization we can use this to <u>approximate</u> the function <u>near</u> the expansion point ($x=1$ in #2 and $x=-8$ in #4). For example,

$$\sqrt[3]{-8.12} \cong -2 + \frac{1}{12}(-8.12+8) = -2 - \frac{0.12}{12} = \boxed{-2.01 \cong \sqrt[3]{-8.12}}$$

After the robot holocaust, when all the calculators are evil, this will be very useful.

§4.1 #2 p. 269 You know that $A = \pi r^2$ for a circle so relate the time-rate of change of Area and radius

Conceptualize that A and r are functions of time. Now differentiate $A = \pi r^2$ with respect to time,

$$\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = \boxed{2\pi r \frac{dr}{dt} = \frac{dA}{dt}}$$

If an oil spill spreads in a circular slick with radius increasing at $1\,m/s$ when $r = 30\,m$ then how quickly is the area of the slick increasing?

$$\frac{dA}{dt} = 2\pi(30m)\left(1\frac{m}{s}\right) = 60\pi\frac{m^2}{s} \cong \boxed{188.5\frac{m^2}{s}}$$

§4.1 #4 p. 269 A particle moves in the (xy)-plane along $y = \sqrt{1+x^3}$. If the y-coordinate is increasing at $4\,cm/s$ as the particle approaches the point $(2,3)$ then how fast does x change then?

Notice we should think of x and y as functions of time. Thus

$$\frac{dy}{dt} = \frac{d}{dt}\left(\sqrt{1+x^3}\right) = \frac{1}{2\sqrt{1+x^3}}\frac{d}{dt}(1+x^3) = \frac{3x^2}{2\sqrt{1+x^3}}\frac{dx}{dt}$$

Now solve for $\frac{dx}{dt}$ and evaluate at the point in question,

$$\left.\frac{dx}{dt}\right|_{(2,3)} = \left.\left(\frac{2\sqrt{1+x^3}}{3x^3}\frac{dy}{dt}\right)\right|_{(2,3)}$$

(Evaluate at the point $(2,3)$)

$$= \frac{2\cancel{\sqrt{1+8}}}{\cancel{3}\cdot 8}\cdot 4\frac{cm}{s}$$

$$= \boxed{1\frac{cm}{s} = \frac{dx}{dt}}$$

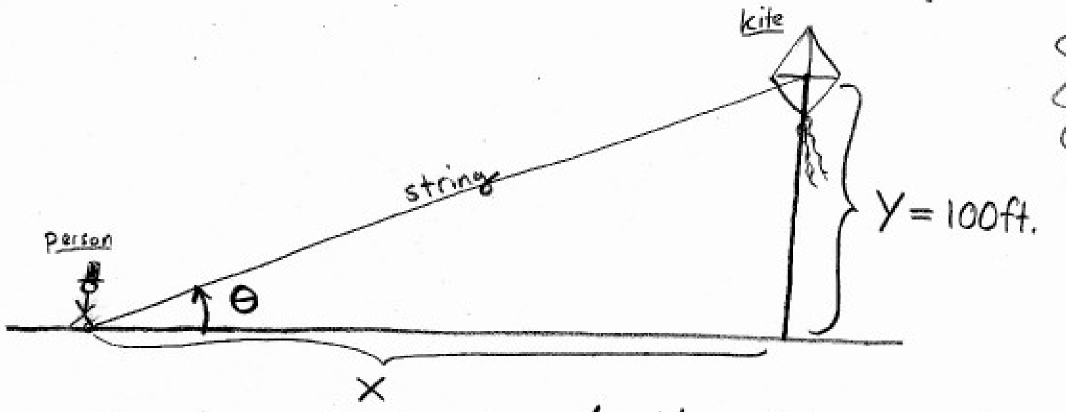
- $\frac{dx}{dt}$ is how fast x changes.

- The book is very careless about units here! if x has cm for units then how can I add it to 1? they take x and y to be unitless, but then give $\frac{dx}{dt}$ units! Most inconsistent! Grrr....

§4.1 # 22 p. 270 — A kite 100ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle θ between the string and the horizontal (ground) decreasing when 200ft of string have been let out.

- Notice this is a silly math kite which <u>stays</u> at $100ft = y$ througout the problem.

We need to draw a picture (as is often the case in these problems)



why don't I put $x = 200ft$ yet?

Here x depends on θ in general. Use trigonometry to see,

$$\tan\theta = \frac{100}{x}$$

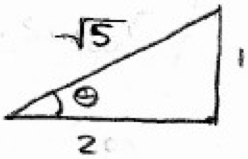
Now θ and x are functions of time, diff. w.r.t. time,

$$\sec^2\theta \frac{d\theta}{dt} = \frac{-100}{x^2}\frac{dx}{dt}$$

Thus we find,

$$\frac{d\theta}{dt} = \frac{-100}{x^2\sec^2\theta}\frac{dx}{dt}$$

We wish to find $\frac{d\theta}{dt}$ when $x = 200ft$. What is $\sec^2\theta$ there?



$$\sec\theta = \frac{\sqrt{5}}{2} \quad \therefore \quad \sec^2\theta = \frac{5}{4}$$

We know that $\frac{dx}{dt} = 8\,ft/s$ so put it together,

$$\left.\frac{d\theta}{dt}\right|_{x=200ft} = \underbrace{\frac{-100ft}{(200ft)^2\cdot\frac{5}{4}}}\cdot 8\frac{ft}{s} = \boxed{-0.16\ \frac{rad}{s}}$$

- need to give x units of ft otherwise we cannot cancel out the ft in $\frac{ft}{s}$. So in this problem we don't consider x & y unitless! grr...

(I technically should have $\tan\theta = \frac{100ft}{x}$ and so on...)