CALCULUS 131:	TEST 1: CONTINUITY AND LIMITS

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks. You are allowed the use of a scientific (non-graphing) calculator. No electronic communication devices of any kind permitted, no IPODs, Zunes, Walkmans etc... This is a timed test and time is likely to be an issue for you, budget your time wisely. There are 150pts to earn on this exam.

Problem 1 [6pts] Simplify the expressions below into the format cx^n where c, n are constants.

$(a.)$ $\dfrac{1}{3\sqrt[5]{x^2}} =$ _____, $(b.)$ $\dfrac{2}{7x^3} =$ _____, $(c.)$ $\sqrt[3]{5x^8} =$ _____.

Problem 2 [4pts] State the domain and range for each of the inverse trigonometric functions:

$(a.)$ $dom(\sin^{-1}(x)) =$ _____,

$(b.)$ $dom(\tan^{-1}(x)) =$ _____,

$(d.)$ $range(\sin^{-1}(x)) =$ _____,

$(f.)$ $range(\tan^{-1}(x)) =$ _____.

Problem 2 [3pts] Suppose that f is a one-one function and $a, b \in dom(f)$. If $f(a) = f(b)$ then is it true that $a = b$? (one sentence should do.)

Problem 3 [7pts] Suppose that $f(x) = x^2$ and $dom(f) = [-1, 1]$ is this function invertible? If it is invertible find its inverse. If it is not invertible then find a restriction of f which is invertible. Please state the domain and range of the inverse (or local inverse) which you find.

Problem 4 [4pts] Solve the inequality $x^2 - 3 < 2x$.

Problem 5 [8pts] Find the domain of $f(x) = 1/\cos(x - \pi/2)$.

Problem 6 [20pts] Prove by the $\epsilon\delta$-definition of the limit that

$$\lim_{x \to -2} (x^2 + 3x - 4) = -6.$$

Problem 7 [3pts] Find all real solutions of $x^3 = x$.

Problem 8 [4pts] Suppose the tangent line to $y = f(x)$ at $x = 2$ has equation $y = 42 + 13(x - 2)$. Fill in the blanks:

$f(2) =$ _____. $f'(2) =$ _____.

Problem 9 [30pts] Find the following limits, if they exist. If they don't exist you may simply state $\pm\infty, d.n.e.$ or similar brief description of the divergent nature of the given limit.

(*a.*) $\lim_{x\to 0^+} \ln(x)$

(*b.*) $\lim_{x\to 3^+} \dfrac{-2}{(3-x)^2}$

(*c.*) $\lim_{x\to 5} \dfrac{\sqrt{x+11}-4}{x-5}$

(*d.*) $\lim_{x\to 2} \dfrac{x-2}{x^3-2x^2+2x-4}$

(e.) $\lim_{h\to 0} \dfrac{2(1+h)^2+3-5}{h}$

Problem 10 [5pts] Given the position $s(t) = 2t^2 + 3$ calculate the velocity at time $t = 1$ (hint: may use previous calculations on test).

Problem 11 [10pts] Show f defined below is discontinuous at $x = 2$.

$$f(x) = \begin{cases} 3|x-2|(x-2)^{-1} & if\ x < 2 \\ 3 & if\ x \geq 2 \end{cases}$$

Problem 12 [10pts] Suppose that $10 + \ln(5-x) \leq f(x) \leq 3\sqrt{x} + x$ for $3 < x < 4.5$. Calculate $\lim_{x\to 4} f(x)$.

Problem 13 [3pts] Let f be a function. State the definition of continuity at a point $a \in int(dom(f))$ (recall int indicates the interior, or inside of the set).

Problem 14 [12pts] Find a value for c such that function below is continuous at $x = 2$. Explain your choice in terms of arguments with limits.

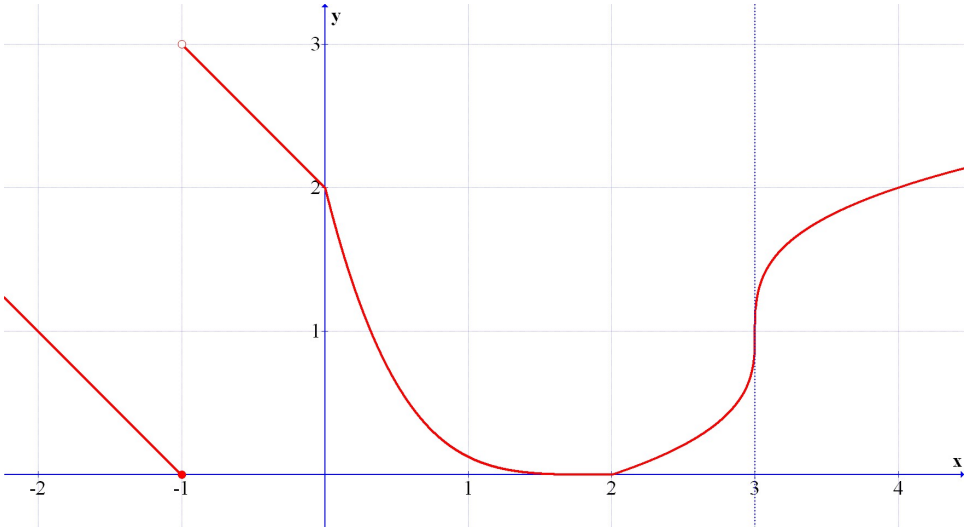
$$f(x) = \begin{cases} cx - 1 & if\ x \leq 2 \\ \frac{4c(x-2)}{x^2-4} & if\ x > 2 \end{cases}$$

Problem 15 [5pts] Use the intermediate value theorem to prove that there exists a solution to the equation below: hint: think about the interval $[0, 4]$.

$$\sqrt{4x} - \sqrt[3]{x/4} = 1$$

Problem 16 [10pts] Let $f(x) = 1/x$. Show, by the definition, that $f'(x) = -1/x^2$ for $x \neq 0$.

Problem 17 [8pts] Read the derivatives from the graph of $y = f(x)$ given below, if possible. If not possible briefly explain why.



$(a.) f'(-1)$ $(b.) f'(0)$ $(c.) f'(1.7)$ $(d.) f'(3)$

Problem 18 [2pts] State the defining formula: $\cosh(x) =$ _____.