§17.6 #22 | Find tangent plane at $(0, \sqrt{2}, 1)$. I'm going to use $\Phi(y,z) = \langle 4 - y^2 - 2z^2, y, z \rangle$

$$\Phi_y = \langle -2y, 1, 0 \rangle$$

$$\Phi_z = \langle -4z, 0, 1 \rangle$$

$$\Phi_y \times \Phi_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2y & 1 & 0 \\ -4z & 0 & 1 \end{vmatrix} = \langle 1, 2y, 4z \rangle = \vec{N}(y,z)$$

$$\vec{N}(\sqrt{2}, 1) = \langle 1, 2\sqrt{2}, 4 \rangle$$

$$\Rightarrow 1(x-0) + 2\sqrt{2}(y - \sqrt{2}) + 4(z-1) = 0$$

$$x + 2\sqrt{2}(y - \sqrt{2}) + 4(z-1) = 0$$

$$x + 2\sqrt{2}\,y + 4z = 2(\sqrt{2})^2 + 4 = 8$$

$$\boxed{x + 2\sqrt{2}\,y + 4z = 8}$$

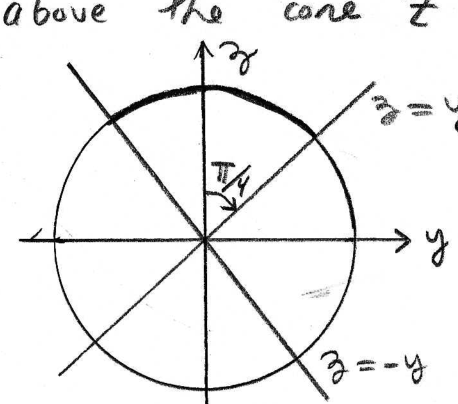
Let's check with level surface idea,

$$F(x,y,z) = x + y^2 + 2z^2 = 4$$

$$\nabla F = \langle 1, 2y, 4z \rangle$$

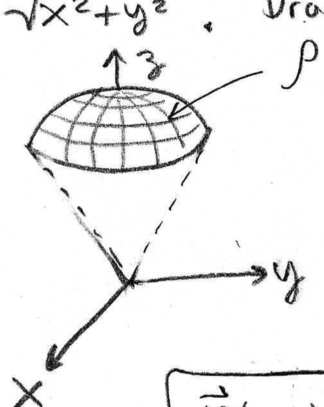
It checks.

§17.6 #23 | The part of sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2+y^2}$. Draw a few pictures.



$X = 0$

$z = \sqrt{0^2 + y^2} = \pm y$



$\rho = 2$ and $0 \le \phi \le \frac{\pi}{4}$ where $0 \le \theta \le 2\pi$. We use θ & φ to parametrize part of a sphere.

$$\boxed{\begin{gathered} \vec{r}(\theta,\varphi) = \langle 2\cos\theta\sin\varphi, 2\sin\theta\sin\varphi, 2\cos\varphi \rangle \\ 0 \le \varphi \le \pi/4, \quad 0 \le \theta \le 2\pi \end{gathered}}$$

§17.6 #20 Parametrize the lower half of the ellipsoid $2x^2+4y^2+z^2 = 1$. 4

We can use modified spherical coordinates, θ and ϕ still work we just need different radii in x, y, z.

$$\left.\begin{aligned} x &= \frac{1}{\sqrt{2}}\cos\theta\sin\varphi \\ y &= \frac{1}{2}\sin\theta\sin\varphi \\ z &= \cos\varphi \end{aligned}\right\} \text{ with } \begin{aligned} &0 \le \theta \le 2\pi \\ &\cancel{0 \le \phi \le \pi/2} \\ &\pi/2 \le \phi \le \pi \end{aligned}$$

$\leftarrow$ just want ~~top~~ half bottom!

You might doubt me, let's check my answer,

$$2x^2 = 2\,\tfrac{1}{2}\cos^2\theta\sin^2\varphi$$
$$4y^2 = 4\,\tfrac{1}{4}\sin^2\theta\sin^2\varphi$$
$$z^2 = \cos^2\varphi$$

$$\begin{aligned} \Rightarrow 2x^2+4y^2+z^2 &= \cos^2\theta\sin^2\varphi + \sin^2\theta\sin^2\varphi + \cos^2\varphi \\ &= \sin^2\varphi(\cos^2\theta+\sin^2\theta) + \cos^2\varphi \\ &= \sin^2\varphi + \cos^2\varphi \\ &= 1. \quad \text{(it works)} \end{aligned}$$

The parametrization is

$$\boxed{\Sigma(\theta,\varphi) = \left\langle \frac{1}{\sqrt{2}}\cos\theta\sin\varphi,\ \frac{1}{2}\sin\theta\sin\varphi,\ \cos\varphi \right\rangle \quad \begin{aligned} &0 \le \theta \le 2\pi \\ &\cancel{0 \le \varphi \le \pi/2} \end{aligned}}$$
$$\pi/2 \le \varphi \le \pi$$

To find the tangent plane at $(1,0,0)$. We have several options,

1.) Could use level surface viewpoint

$$F(x,y,z) = 2x^2+4y^2+z^2 = 1$$
$$\nabla F = \langle 4x, 8y, 2z \rangle$$
$$\nabla F(1,0,0) = \langle 4, 0, 0 \rangle$$

Thus the tangent plane is $4(x-1) = 0$

which is simply $\boxed{x=1}$

(this will look nice in Mathematica)