DERIVATIVES HOMEWORK SOL'NS

§3.1 #'s 4 - 22 p. 197

DIFFERENTIATE THE FUNCTIONS BELOW $f'(x) = \frac{df}{dx} = \frac{d}{dx}(f(x))$

4.) $F'(x) = \frac{d}{dx}\left(-4x^{10}\right) = -4\frac{d}{dx}\left(x^{10}\right) = -4 \cdot 10x^{10-1} = \boxed{-40x^9}$

6.) $g'(x) = \frac{d}{dx}\left(5x^8 - 2x^5 + 6\right) = 5 \cdot 8x^7 - 2 \cdot 5x^4 = \boxed{40x^7 - 10x^4}$

8.) $\frac{dy}{dx} = \frac{d}{dx}\left(5e^x + 3\right) = 5 \cdot \frac{d}{dx}\left(e^x\right) + \underbrace{\frac{d}{dx}(3)}_{0} = \boxed{5e^x}$

10.) $R'(t) = \frac{d}{dt}\left(5t^{-3/5}\right) = 5 \cdot \left(\frac{-3}{5}t^{-\frac{3}{5}-1}\right) = \boxed{-3t^{-\frac{8}{5}}}$

12.) $R'(x) = \frac{d}{dx}\left(\frac{\sqrt{10}}{x^7}\right) = \sqrt{10}\frac{d}{dx}\left(x^{-7}\right) = \sqrt{10} \cdot (-7x^{-8}) = \boxed{\frac{-7\sqrt{10}}{x^8}}$

14.) $\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x}(x-1)\right) = \frac{d}{dx}\left(x^{3/2} - x^{1/2}\right) = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-\frac{1}{2}} = \boxed{\frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}}$

16.) $H'(s) = \frac{d}{ds}\left(\left[\frac{s}{2}\right]^5\right) = \frac{d}{ds}\left(s^5 \cdot \left(\frac{1}{2}\right)^5\right) = \left(\frac{1}{2}\right)^5 \frac{d}{ds}\left(s^5\right) = \boxed{\frac{5s^4}{32}}$

18.) $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2 - 2\sqrt{x}}{x}\right) = \frac{d}{dx}\left(x - 2\frac{\sqrt{x}}{x}\right) = \frac{d}{dx}\left(x - 2x^{-\frac{1}{2}}\right) = 1 - 2\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$

$= \boxed{1 + x^{-3/2}}$

20.) $\frac{dy}{dv} = \frac{d}{dv}\left(ae^v + \frac{b}{v} + \frac{c}{v^2}\right)$

$= a\frac{d}{dv}\left(e^v\right) + b\frac{d}{dv}\left(\frac{1}{v}\right) + c\frac{d}{dv}\left(\frac{1}{v^2}\right)$

$= \boxed{ae^v - b\frac{1}{v^2} - 2c\frac{1}{v^3}}$

• technically the question was ambiguous. Which letter should be the variable, a, b, c or v.? You could just as correctly differentiate with respect to a, b or c the way the question is asked. For example if v is constant, $\frac{dy}{da} = e^v$ $\frac{dy}{db} = \frac{1}{v}$ $\frac{dy}{dc} = \frac{1}{v^2}$

22.) $\frac{du}{dt} = \frac{d}{dt}\left(\sqrt[3]{t^2} + 2\sqrt{t^3}\right)$

$= \frac{d}{dt}\left(t^{2/3} + 2t^{3/2}\right)$

$= \frac{2}{3}t^{-1/3} + 2 \cdot \frac{3}{2}t^{1/2} = \boxed{\frac{2}{3\sqrt[3]{t}} + 3\sqrt{t}}$

§ 3.1 #34 p. 197

Find the equation of tangent line to the curve $y = x^2 + 2e^x$ at the point $(0, 2)$ then graph the curve and that tangent line

Lets find the derivative to begin

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + 2e^x\right) = \frac{d}{dx}\left(x^2\right) + 2\frac{d}{dx}\left(e^x\right) = 2x + 2e^x = 2(x + e^x)$$

What is the derivative at the point in question? The point is $(0, 2)$ so $x = 0$ thus,

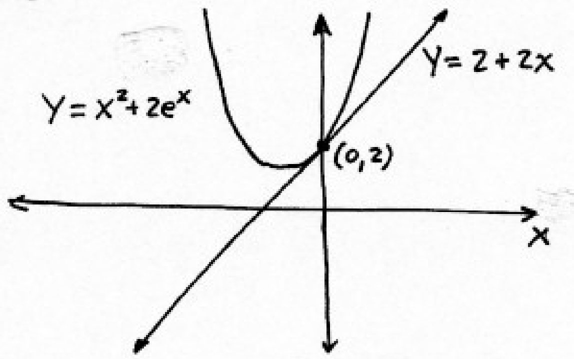
$$\left.\frac{dy}{dx}\right|_{x=0} = 2(0 + e^0) = 2$$

slope of tangent line thru $(0,2)$ is given by derivative $\frac{dy}{dx}$ evaluated at the point.

Now we need a line thru the point $(0,2)$ with slope $m = 2$ use point-slope form $y = y_o + m(x - x_o)$ to find,

$$y = 2 + 2(x - 0) = \boxed{2 + 2x = y \ : \ Eq^n \text{ of Tangent line}}$$

Lets graph the result,



§ 3.1 #42 p. 197

We define the position to be s, the velocity is v, the acceleration is a these are all functions of time t and are related by

$$v = \frac{ds}{dt} \quad \text{and} \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$$

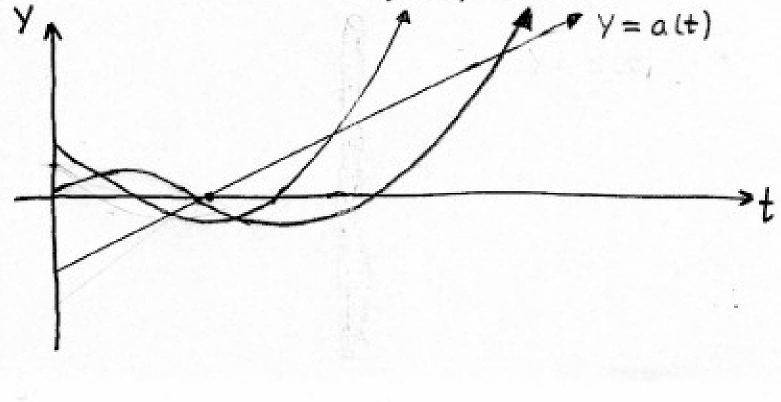
Now suppose that $s(t) = 2t^3 - 7t^2 + 4t + 1$ where s is in meters and t is in seconds. (a.) find $v(t)$ and $a(t)$, (b.) find $a(1)$, (c.) graph s, v and a

a.) $v = \frac{ds}{dt} = \frac{d}{dt}\left(2t^3 - 7t^2 + 4t + 1\right) = \boxed{6t^2 - 14t + 4 = v(t)}$

$a = \frac{dv}{dt} = \frac{d}{dt}\left(6t^2 - 14t + 4\right) = \boxed{12t - 14 = a(t)}$

b.) acceleration at $t = 1$ is $a(1) = 12(1) - 14 = \boxed{-2 = a(1)}$

c.)



§ 3.1 # 46 p. 197	For what values of x does the graph $Y = f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

A horizontal tangent has slope zero. Thus look for sol^{n}'s of $f'(x) = 0$

$$f'(x) = \frac{d}{dx}\left(2x^3 - 3x^2 - 6x + 87\right) = 6x^2 - 6x - 6 = 0$$

We need to solve $6x^2 - 6x - 6 = 0$ aka. $x^2 - x - 1 = 0$. Use the quadratic formula, $ax^2 + bx + c = 0$ when,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \boxed{\frac{1 \pm \sqrt{5}}{2} = x}$$

§ 3.1 # 50 p. 198	Find the two tangent lines to $Y = x^2 + x$ which pass thru the point $(2, -3)$

$$\frac{dY}{dx} = \frac{d}{dx}\left(x^2 + x\right) = 2x + 1$$

The tangent lines have slope given by the above (For certain values of x lets call them "A" for the moment). We also know the tangents pass thru $(2, -3)$. Use point-slope form where here the slope is a variable.

$$Y = Y_0 + m(x - x_0) = \boxed{-3 + (2A + 1)(x - 2) = Y}$$

Eq^{n} of Tangent Line(s)

We need to figure out what A should be. What else do we know? We know the tangent intersects the curve $Y = x^2 + x$ at the unknown $x = A$,

$$Y_{curve} = Y_{tangent} \quad (\text{when } x = A)$$

$$A^2 + A = -3 + (2A + 1)(A - 2)$$

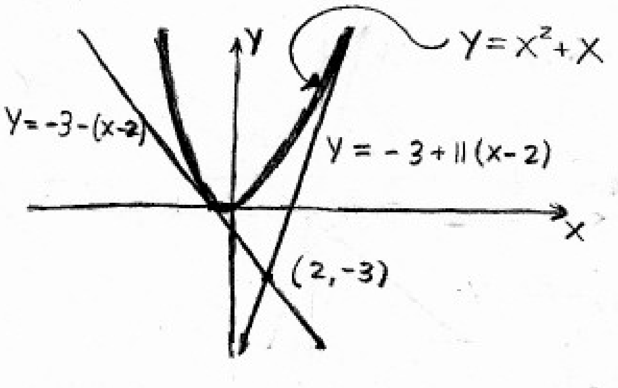
$$A^2 + \cancel{A} = -3 + 2A^2 - 4A + \cancel{A} - 2$$

$$A^2 - 4A - 5 = 0$$

$$(A - 5)(A + 1) = 0 \quad \therefore \quad A = 5 \text{ OR } A = -1$$

Thus the tangent lines are,

$$\boxed{Y = -3 + 11(x - 2)}$$
$$\boxed{Y = -3 - (x - 2)}$$



§3.1 #55 p. 198 — Find 2^{nd} degree polynomial such that $P(2) = 5$, $P'(2) = 3$ and $P''(2) = 2$

We know $P(x) = Ax^2 + Bx + C$ with $A \neq 0$. Our job is to find A, B and C.

$$P'(x) = 2Ax + B$$
$$P''(x) = 2A$$

Lets use what we know

$$P''(2) = 2 = 2A \quad \therefore \quad \boxed{A = 1}$$

$$P'(2) = 2A(2) + B = 3 \Rightarrow 4 + B = 3 \Rightarrow \boxed{B = -1}$$

$$P(2) = A(2)^2 + B(2) + C = 4 - 2 + C = 5 \Rightarrow \boxed{C = 3}$$

Thus $\boxed{P(x) = x^2 - x + 3}$ is the one we want.

§3.1 #56 p. 198 — The eq^n $y'' + y' - 2y = x^2$ is a differential eq^n because it is an equation which involves unknown y, y' and y''. Guess that $y = Ax^2 + Bx + C$ satisfies the diff. eq^n and find A, B, C that make y a sol^n.

$$y = Ax^2 + Bx + C$$
$$y' = 2Ax + B$$
$$y'' = 2A$$

If y is a sol^n then it makes the eq^n true when we substitute the function $y = Ax^2 + Bx + C$ into the eq^n.

$$y'' + y' - 2y = x^2$$

$$2A + 2Ax + B - 2(Ax^2 + Bx + C) = x^2$$

$$x^2(1 + 2A) + x(-2A + 2B) - 2A - B + 2C = 0$$

This means that each coefficient of the polynomial is zero, ("This is called equating coefficient")

$$1 + 2A = 0 \Rightarrow \boxed{A = -1/2}$$

$$2B - 2A = 0 \Rightarrow 2B + 1 = 0 \Rightarrow \boxed{B = -1/2}$$

$$2C - 2A - B = 0 \Rightarrow 2C + 1 + \tfrac{1}{2} = 0 \Rightarrow \boxed{C = \tfrac{-3}{4}}$$

Thus $\boxed{y = \frac{-1}{2}x^2 - \frac{1}{2}x - \frac{3}{4}}$ is a sol^n to the diff. eq^n above.

BONUS POINT: If $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$ for all x. Show that all the coefficients are zero. Use differentiation to prove it. (Hint, try putting $x = 0$ after differentiating)

§3.2 # 4 → 10 p. 204	Use the product & quotient rules to calculate the derivatives below.

4.) $$\frac{d}{dx}\left(\sqrt{x}\,e^x\right) = \frac{d}{dx}\left(\sqrt{x}\right)e^x + \sqrt{x}\,\frac{d}{dx}\left(e^x\right)$$: product rule.

$$= \boxed{\frac{1}{2\sqrt{x}}\,e^x + \sqrt{x}\,e^x}$$: $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

6.) $$\frac{d}{dx}\left(\frac{e^x}{1+x}\right) = \frac{\frac{d}{dx}(e^x)\,(1+x) - e^x\frac{d}{dx}(1+x)}{(1+x)^2}$$: quotient rule

$$= \frac{e^x(1+x) - e^x}{(1+x)^2}$$

$$= \boxed{\frac{xe^x}{(1+x)^2}}$$

8.) $$\frac{d}{du}\left(\frac{1-u^2}{1+u^2}\right) = \frac{\frac{d}{du}(1-u^2)\cdot(1+u^2) - (1-u^2)\frac{d}{du}(1+u^2)}{(1+u^2)^2}$$: quotient rule.

$$= \frac{-2u(1+u^2) - (1-u^2)\cdot 2u}{(1+u^2)^2}$$

$$= \frac{-2u - \cancel{2u^3} - 2u + \cancel{2u^3}}{(1+u^2)^2}$$

$$= \boxed{\frac{-4u}{(1+u^2)^2}}$$

10.) $$\frac{d}{dt}\left(e^t[1+3t^2+5t^4]\right) = \left(\frac{de^t}{dt}\right)\cdot[1+3t^2+5t^4] + e^t\frac{d}{dt}[1+3t^2+5t^4]$$

$$= e^t[1+3t+5t^4] + e^t[6t+20t^3]$$

$$= \boxed{e^t[1+9t+20t^3+5t^4]}$$

§3.2 # 12–18 p. 204	Differentiate using: $(fg)' = f'g + fg'$ & $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

12.) $$\frac{d}{dt}\left(\frac{t^3+t}{t^4-2}\right) = \frac{(t^3+t)'(t^4-2) - (t^3+t)(t^4-2)'}{(t^4-2)^2}$$: notice the notation, $(t^3+t)' \equiv \frac{d}{dt}(t^3+t)$

$$= \frac{(3t^2+1)(t^4-2) - (t^3+t)(4t^3)}{(t^4-2)^2}$$

$$= \frac{3t^6 - 6t^2 + t^4 - 2 - (4t^6 + 4t^4)}{(t^2-2)^2}$$

$$= \boxed{\frac{-t^6 - 3t^4 - 6t^2 - 2}{(t^2-2)^2}}$$

14.) $$\frac{d}{ds}\left(\frac{1}{s+ke^s}\right) = \frac{\cancel{(1)'(s+ke^s)}^{\,0} - 1\cdot\frac{d}{ds}(s+ke^s)}{(s+ke^s)^2}$$

$$= \boxed{\frac{-1+ke^s}{(s+ke^s)^2}}$$

16.) $$\frac{d}{dw}\left(w^{3/2}(w+ce^w)\right) = \frac{d}{dw}(w^{3/2})\cdot(w+ce^w) + w^{3/2}\frac{d}{dw}(w+ce^w)$$

$$= \boxed{\frac{3}{2}w^{1/2}(w+ce^w) + w^{3/2}(1+ce^w)}$$

18.) $$\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right) = \frac{\frac{d}{dx}(ax+b)\cdot(cx+d) - (ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2}$$

$$= \frac{a(cx+d) - (ax+b)c}{(cx+d)^2}$$

$$= \frac{\cancel{acx} + ad - \cancel{acx} - bc}{(cx+d)^2}$$

$$= \boxed{\frac{ad-bc}{(cx+d)^2}}$$

• Remark: We assumed that a, b, c, d, and k were constants in problems 14, 16 and 18. A constant has derivative zero because it does not change!

§3.2 #20 p. 204	$f(x) = \frac{\sqrt{x}}{x+1}$ find tangent line thru (4, 0.4)

$$f'(x) = \frac{\frac{d}{dx}(\sqrt{x})(x+1) - \sqrt{x}\frac{d}{dx}(x+1)}{(x+1)^2} = \frac{1}{(x+1)^2}\left(\frac{1}{2\sqrt{x}}(x+1) - \sqrt{x}\right)$$

Thus we find, $f'(4) = \frac{1}{5^2}\left(\frac{1}{2\sqrt{4}}(4+1) - \sqrt{4}\right) = \frac{1}{25}\left(\frac{5}{4} - 2\right) = \frac{-3}{100}$. So the tangent line to $y = f(x)$ thru $(4, 0.4)$ has slope $-3/100$. Using point-slope form of line we get

$$\boxed{y = 0.4 - \frac{3}{100}(x-4)}$$

§3.2 #30 p. 205	If $h(2) = 4$ and $h'(2) = -3$ find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}$

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right) = \frac{\frac{dh}{dx}x - h\frac{dx}{dx}}{x^2} = \frac{1}{x^2}\left(xh'(x) - h(x)\right)$$

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2} = \frac{1}{4}\left(2h'(2) - h(2)\right) = \frac{1}{4}\left(2(-3) - 4\right) = \frac{-10}{4} = \boxed{\frac{-5}{2}}$$

§3.4 #2-6 p. 223	Differentiate. Use product rule and your knowledge of basic derivatives.

2.) $\frac{d}{dx}\left(x\sin(x)\right) = \frac{dx}{dx}\sin(x) + x\frac{d}{dx}\left(\sin(x)\right)$: product rule

$= \boxed{\sin(x) + x\cos(x)}$

4.) $\frac{d}{dt}\left(4\sec t + \tan t\right) = 4\frac{d}{dt}\left(\sec(t)\right) + \frac{d}{dt}\left(\tan(t)\right)$

$= \boxed{4\sec(t)\tan(t) + \sec^2(t)}$

6.) $\frac{d}{du}\left(e^u(\cos u + cu)\right) = \frac{de^u}{du}(\cos u + cu) + e^u\frac{d}{du}\left(\cos u + cu\right)$: product rule.

$= e^u(\cos u + cu) + e^u(-\sin u + c)$: c is a constant.

$= \boxed{e^u(\cos u - \sin u + cu + c)}$

§3.4 # 8 − 14 p. 223	Differentiate use what you know, if you don't know it then learn it.

8.) $$\frac{d}{dx}\left(\frac{\sin(x)}{1+\cos(x)}\right) = \frac{\frac{d}{dx}(\sin(x))\cdot(1+\cos(x)) - \sin(x)\cdot\frac{d}{dx}(1+\cos(x))}{(1+\cos(x))^2}$$

$$= \frac{\cos(x)(1+\cos(x)) - \sin(x)(-\sin(x))}{(1+\cos(x))^2}$$

$$= \frac{\cos(x) + \cos^2(x) + \sin^2(x)}{(1+\cos(x))^2}$$

$$= \frac{\cos(x)+1}{(1+\cos(x))^2}$$

$$= \boxed{\frac{1}{1+\cos(x)}}$$

10.) $$\frac{d}{dx}\left(\frac{\tan(x)-1}{\sec(x)}\right) = \frac{d}{dx}\left(\frac{\tan(x)}{\sec(x)} - \frac{1}{\sec(x)}\right)$$

$$= \frac{d}{dx}\left(\sin(x) - \cos(x)\right)$$

$$= \boxed{\cos(x) + \sin(x)}$$

$\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$

12.) $$\frac{d}{d\theta}\left(\csc\theta\,(\theta+\cot\theta)\right) = \frac{d}{d\theta}(\csc\theta)\cdot(\theta+\cot\theta) + \csc\theta\,\frac{d}{d\theta}(\theta+\cot\theta)$$

$$= -\csc\theta\cot\theta\,(\theta+\cot\theta) + \csc\theta\,(1-\csc^2\theta)$$

$$= \csc\theta\left[-\theta\cot\theta - \cot^2\theta + 1 - \csc^2\theta\right]$$

$$= \csc\theta\left[-\theta\cot\theta - 2\cot^2\theta\right]$$

$$= \boxed{\csc\theta\cot\theta\left[-\theta - 2\cot\theta\right]}$$

14.) $$\frac{d}{dx}\left(\sec(x)\right) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{(1)'\cos(x) - 1\,(\cos(x))'}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos(x)\cos(x)}$$

$$= \boxed{\sec(x)\tan(x)}$$

- Here we have proved that $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

In other problems you can use this, (like #10, except I did #10 a sneaky way ☺).

§3.4 #18 p. 223	$y = e^x\cos(x)$ find tangent line thru $(0,1)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x\cos(x)\right) = \frac{d}{dx}\left(e^x\right)\cdot\cos(x) + e^x\frac{d}{dx}\left(\cos(x)\right) = e^x\left(\cos(x) - \sin(x)\right)$$

So we find $\left.\frac{dy}{dx}\right|_{x=0} = e^0(\cos(0) - \sin(0)) = 1$. Then using point-slope form of line to find line with $m=1$ and $y(0)=1$,

$$y = 1 + 1(x-0) = \boxed{x+1 = y}$$

§3.4 #26 p. 224	find the values of x for which $y = \frac{\cos(x)}{2+\sin(x)}$ has a horizontal tangent

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos(x)}{2+\sin(x)}\right) = \frac{(\cos(x))'(2+\sin(x)) - \cos(x)(2+\sin(x))'}{(2+\sin(x))^2}$$

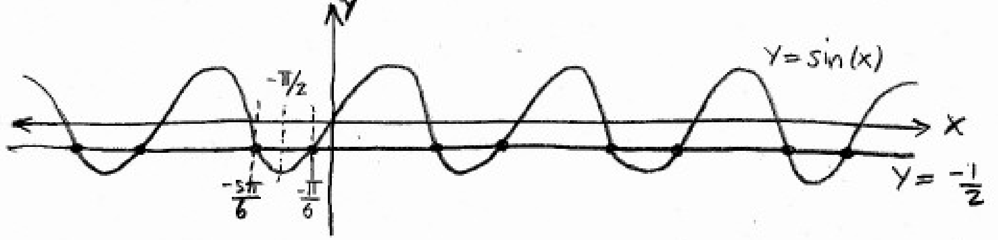
$$= \frac{-\sin(x)(2+\sin(x)) - \cos(x)\cos(x)}{(2+\sin(x))^2}$$

$$= \frac{-2\sin(x) - \sin^2(x) - \cos^2(x)}{(2+\sin(x))^2}$$

Horizontal Tangents: $\frac{dy}{dx} = 0$. Now we need the numerator to be zero (If the demominator is zero that does <u>not</u> give a sol$^{\underline{n}}$!)

$$-2\sin(x) - \sin^2(x) - \cos^2(x) = -2\sin(x) - 1 = 0$$

Hence $\sin(x) = \frac{-1}{2}$ gives the horizontal tangents. How do we solve such an equation? By graph is one way



$$\sin\left(\frac{-\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

By symmetry you can see that the other sol$^{\underline{n}}$ is an equal distance from $\frac{-\pi}{2}$. Notice $\frac{-\pi}{2}$ is $\frac{2\pi}{6}$ away from $\frac{-\pi}{6}$ thus

$-\frac{\pi}{2} - \frac{2\pi}{6} = -\frac{5\pi}{6}$ is the other sol$^{\underline{n}}$, $\sin\left(\frac{-5\pi}{6}\right) = \frac{-1}{2}$.

Now periodicity of sine gives the rest of the sol$^{\underline{n}}$'s

$\sin(x) = \frac{-1}{2}$ for $\boxed{x = \frac{-\pi}{6} + 2\pi n \text{ and } -\frac{5\pi}{6} + 2\pi n \text{ for } n \in \mathbb{Z}}$

§3.5 #2–6 p. 233	Explicitly identify the structure of the composite functions as $f(g(x))$ where $u = g(x)$. Then find $\frac{d}{dx}(f(g(x)))$

2.) $Y = \sqrt{4+3x} = f(g(x))$ if we let $\rightarrow f(u) = \sqrt{u}$
$\qquad g(x) = 4+3x = u$

$$\frac{dy}{dx} = \frac{df}{du}\frac{du}{dx} = \frac{d}{du}\left(\sqrt{u}\right)\frac{d}{dx}\left(4+3x\right)$$
$$= \frac{1}{2\sqrt{u}} \cdot 3$$
$$= \boxed{\frac{3}{2\sqrt{4+3x}}}$$

4.) $Y = \tan(\sin(x)) = f(g(x))$ if we let $\longrightarrow f(u) = \tan(u)$
$\qquad g(x) = \sin(x) = u$

$$\frac{dy}{dx} = \frac{df}{du}\frac{du}{dx} = \sec^2(u)\frac{d}{dx}\left(\sin(x)\right)$$
$$= \boxed{\sec^2(\sin(x))\cos(x)}$$

- Notice that $\frac{df}{du}$ implicitly indicates we should evaluate the outer derivative at the inside function u. To be more careful we probably should write $\left.\frac{df}{du}\right|_{u(x)}$ or use $\frac{dy}{dx} = \left.\frac{df}{dx}\right|_{g(x)}\frac{dg}{dx}$. The important thing is you understand how to use the chain rule.

6.) $Y = \sin(e^x) = f(g(x))$ if $f(x) = \sin(x)$ and $g(x) = e^x$ then using the prime notation,

$$\frac{dy}{dx} = (f \circ g)'(x)$$
$$= f'(g(x))\, g'(x)$$
$$= \boxed{\cos(e^x)\, e^x}$$

§3.5 # 8–18 p. 233	Use the Chain Rule to calculate the derivatives of the composite functions below.

8.) $\frac{dF}{dx} = \frac{d}{dx}(x^2 - x + 1)^3 = 3\cdot(x^2 - x + 1)^2 \frac{d}{dx}(x^2 - x + 1)$

$= 3(x^2 - x + 1)^2 (2x - 1)$

$= \boxed{3(2x-1)(x^2 - x + 1)^2}$

10.) $\frac{df}{dt} = \frac{d}{dt}\left(\sqrt[3]{1 + \tan t}\right) = \frac{d}{dt}\left([1 + \tan(t)]^{\frac{1}{3}}\right)$

$= \frac{1}{3}[1 + \tan(t)]^{-\frac{2}{3}} \cdot \frac{d}{dt}(1 + \tan(t))$

$= \boxed{\frac{\sec^2(t)}{3(1 + \tan(t))^{2/3}}}$

12.) $\frac{dy}{dx} = \frac{d}{dx}\left(\underset{0}{a^3} + \cos^3(x)\right) = \frac{d}{dx}\left([\cos(x)]^3\right)$

$= 3(\cos(x))^2 \frac{d}{dx}(\cos(x))$

$= \boxed{-3\cos^2(x)\sin(x)}$

14.) $\frac{d}{dx}(4\sec(5x)) = 4\sec(5x)\tan(5x)\frac{d}{dx}(5x)$

$= \boxed{20\sec(5x)\tan(5x)}$

16.) $\frac{d}{dx}\left(e^{-5x}\cos(3x)\right) = \frac{d}{dx}\left(e^{-5x}\right)\cos(3x) + e^{-5x}\frac{d}{dx}(\cos(3x))$

$= \left[e^{-5x}\frac{d}{dx}(-5x)\right]\cos(3x) + e^{-5x}\left[-\sin(3x)\frac{d}{dx}(3x)\right]$

$= \boxed{-5e^{-5x}\cos(3x) - 3e^{-5x}\sin(3x)}$

18.) $\frac{d}{dt}\left((6t^2+5)^3(t^3-7)^4\right) = \frac{d}{dt}(6t^2+5)^3 \cdot (t^3-7)^4 + (6t^2+5)^3\frac{d}{dt}(t^3-7)^4$

$= \left[3(6t^2+5)^2\frac{d}{dt}(6t^2+5)\right]\cdot(t^3-7)^4 + (6t^2+5)^3\left[4(t^3-7)^3\cdot\frac{d}{dt}(t^3-7)\right]$

$= 3(6t^2+5)^2(12t)(t^3-7)^4 + (6t^2+5)^3 4(t^3-7)^3(3t^2)$

$= \boxed{(6t^2+5)^2(t^3-7)^3 t\left[36(t^3-7) + 12t^2(6t^2+5)\right]}$

§ 3.5 # 20 – 26 p. 233	Differentiate using the chain-rule and whatever else necessary

20.) $\frac{d}{dx}\left(10^{1-x^2}\right) = \frac{d}{dx}\left(10^u\right)$: where $u = 1-x^2$

$= \ln(10)\, 10^u \frac{du}{dx}$

$= \boxed{-2x\ln(10)\, 10^{1-x^2}}$

Sometimes it is helpful to write u down. Sometimes I'll ask you to explicitly explain what you're doing (like in #2, 4, 6)

22.) $\frac{d}{dt}\left(\sqrt[4]{\frac{t^3+1}{t^3-1}}\right) = \frac{d}{dt}\left(\left[\frac{t^3+1}{t^3-1}\right]^{\frac{1}{4}}\right)$

$= \frac{1}{4}\left[\frac{t^3+1}{t^3-1}\right]^{-3/4} \frac{d}{dt}\left(\frac{t^3+1}{t^3-1}\right)$

$= \boxed{\frac{1}{4}\left[\frac{t^3+1}{t^3-1}\right]^{\frac{-3}{4}}\left(\frac{3t^2(t^3-1)-(t^3+1)\,3t^2}{(t^3-1)^2}\right)}$: quotient rule.

(unsimplified answer.)

24.) $\frac{d}{du}\left(\frac{e^{2u}}{e^u+e^{-u}}\right) = \frac{\frac{d}{du}(e^{2u})(e^u+e^{-u}) - e^{2u}\frac{d}{du}(e^u+e^{-u})}{(e^u+e^{-u})^2}$: quotient rule.

$= \frac{1}{(e^u+e^{-u})^2}\left[2e^{2u}(e^u+e^{-u}) - e^{2u}(e^u-e^{-u})\right]$: chain rule several times.

$= \boxed{\frac{e^{2u}}{(e^u+e^{-u})^2}\left[e^u + 3e^{-u}\right]}$

26.) $\frac{d}{d\theta}\left(\tan^2(3\theta)\right) = 2\tan(3\theta)\frac{d}{d\theta}(\tan(3\theta))$: chain rule

$= 2\tan(3\theta)\sec^2(3\theta)\cdot\frac{d}{d\theta}(3\theta)$: chain rule <u>again</u>.

$= \boxed{6\tan(3\theta)\sec^2(3\theta)}$

§3.5 #28-30 p. 233	More chain rule problems Outside in.

28.) $\frac{d}{dx}\left[\sin(\sin(\sin(x)))\right] = \cos(\sin(\sin(x)))\frac{d}{dx}\left(\sin(\sin(x))\right)$

$= \cos(\sin(\sin(x)))\cos(\sin(x))\frac{d}{dx}(\sin(x))$

$= \boxed{\cos(\sin(\sin(x)))\cos(\sin(x))\cos(x)}$

30.) $\frac{d}{dx}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}\right] = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\frac{d}{dx}\left(x+\sqrt{x+\sqrt{x}}\right)$

$= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\frac{d}{dx}\left(x+\sqrt{x}\right)\right)$

$= \boxed{\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right)}$

§3.5 #48 p. 234	If g is differentiable and $f(x) = x\,g(x^2)$ then find f'' in terms of g, g' and g''.

$f'(x) = \frac{d}{dx}\left(x\,g(x^2)\right)$

$= \frac{dx}{dx}g(x^2) + x\frac{d}{dx}\left(g(x^2)\right)$

$= g(x^2) + x\,g'(x^2)\frac{d}{dx}(x^2)$

$= g(x^2) + 2x^2 g'(x^2)$

$f''(x) = \frac{d}{dx}\left(f'(x)\right)$

$= \frac{d}{dx}\left(g(x^2) + 2x^2 g'(x^2)\right)$

$= g'(x^2)\frac{d}{dx}(x^2) + 2\left[\frac{d}{dx}(x^2)\,g'(x^2) + x^2\frac{d}{dx}\left(g'(x^2)\right)\right]$

$= 2x\,g'(x^2) + 2\left[2x\,g'(x^2) + x^2 g''(x^2)\frac{d}{dx}(x^2)\right]$

$= \boxed{6x\,g'(x^2) + 4x^3 g''(x^2)}$

§3.5 #52 p. 234	For what values of r does $y = e^{rx}$ satisfy $y'' + 5y' - 6y = 0$?

$$y = e^{rx}$$
$$y' = re^{rx}$$
$$y'' = r^2 e^{rx}$$

Substitute into $y'' + 5y' - 6y = 0$ to find,

$$r^2 \cancel{e^{rx}} + 5r\cancel{e^{rx}} - 6\cancel{e^{rx}} = 0$$

Why is ok to cancel the e^{rx} ?

$$\Rightarrow \quad r^2 + 5r - 6 = 0$$
$$\Rightarrow (r+6)(r-1) = 0 \qquad \therefore \quad \boxed{r = -6 \text{ or } r = 1}$$

§3.5 #56 p. 234	Let $s = A\cos(\omega t + \delta)$ describe the position s at time t of some particle. Find velocity at time t
and then determine the initial velocity.	

$$v = \frac{ds}{dt} = \frac{d}{dt}\Big(A\cos(\omega t + \delta)\Big)$$
$$= -A\sin(\omega t + \delta)\frac{d}{dt}(\omega t + \delta)$$
$$= \boxed{-A\omega \sin(\omega t + \delta) = v(t)}$$

ω, δ, A are assumed to be constants.

A = amplitude
ω = angular velocity
δ = phase

Then $\boxed{v(0) = -A\omega \sin(\delta)}$.

§3.5 #61 p. 234	A particle moves along a straight line with position s and velocity v and acceleration a.
Show	that $a = v(t)\frac{dv}{ds}$.

$$a = \frac{dv}{dt} = \frac{ds}{dt}\frac{dv}{ds} = v\frac{dv}{ds}$$

Realize that the above is just the chain-rule,

$$\frac{df}{dx} = \frac{d}{dx}\Big(f(u(x))\Big) = \frac{df}{du}\frac{du}{dx} \longleftrightarrow \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} \qquad \begin{pmatrix} x \to t \\ f \to v \\ u \to s \end{pmatrix}$$

$$\therefore \quad \boxed{a = \frac{dv}{dt} = v\frac{dv}{ds}}$$

- $\frac{dv}{dt}$ gives how v changes when t changes.
- $\frac{dv}{ds}$ gives how v changes when s changes.

§3.6 #4–8 p. 243	Find $\frac{dy}{dx}$ by implicit differentiation

4.) $x^2 - 2xy + y^3 = c$

Differentiate both sides. Remember $y = y(x)$ (it is a function of x)

$$2x - 2\left(\frac{dx}{dx}y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$ (used product & chain)

$$2x - 2y - 2x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3y^2 - 2x) = 2y - 2x \Rightarrow \boxed{\frac{dy}{dx} = \frac{2y-2x}{3y^2-2x}}$$

6.) $y^5 + x^2y^3 = 1 + ye^{x^2}$

$$\frac{d}{dx}(y^5 + x^2y^3) = \frac{d}{dx}(1 + ye^{x^2})$$

$$5y^4\frac{dy}{dx} + 2xy^3 + x^2 \cdot 3y^2\frac{dy}{dx} = \frac{dy}{dx}e^{x^2} + ye^{x^2} \cdot (2x)$$

$$\frac{dy}{dx}\left(5y^4 + 3x^2y^2 - e^{x^2}\right) = 2xye^{x^2} - 2xy^3$$

$$\boxed{\frac{dy}{dx} = \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}}$$

8.) $\sqrt{1+x^2y^2} = 2xy$

$$\frac{d}{dx}\sqrt{1+x^2y^2} = \frac{d}{dx}(2xy)$$

$$\frac{1}{2\sqrt{1+x^2y^2}}\frac{d}{dx}(1+x^2y^2) = 2\left(\frac{dx}{dx}y + x\frac{dy}{dx}\right)$$

$$\frac{1}{2\sqrt{1+x^2y^2}}\left(2xy^2 + x^2 \cdot 2y\frac{dy}{dx}\right) = 2y + 2x\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{x^2y}{\sqrt{1+x^2y^2}} - 2x\right) = 2y - \frac{xy^2}{\sqrt{1+x^2y^2}}$$

$$\frac{dy}{dx} = \frac{2y - \frac{xy^2}{\sqrt{1+x^2y^2}}}{\frac{x^2y}{\sqrt{1+x^2y^2}} - 2x} = \boxed{\frac{2y\sqrt{1+x^2y^2} - xy^2}{x^2y - 2x\sqrt{1+x^2y^2}} = \frac{dy}{dx}}$$

§3.6 # 10–12 p. 243	find $\frac{dy}{dx}$ by implicit differentiation

10.) $x\cos(y) + y\cos(x) = 1$

$$\frac{d}{dx}\left(x\cos(y) + y\cos(x)\right) = \frac{d}{dx}(1) = 0$$

$$\cos(y) - \sin(y)\frac{dy}{dx} + \frac{dy}{dx}\cos(x) - y\sin(x) = 0$$

$$\frac{dy}{dx}\left(\cos(x) - \sin(y)\right) = y\sin(x) - \cos(y)$$

$$\boxed{\frac{dy}{dx} = \frac{y\sin(x) - \cos(y)}{\cos(x) - \sin(y)}}$$

12.) $\sin(x) + \cos(y) = \sin(x)\cos(y)$

$$\frac{d}{dx}\left(\sin(x) + \cos(y)\right) = \frac{d}{dx}\left(\sin(x)\cos(y)\right)$$

$$\cos(x) - \sin(y)\frac{dy}{dx} = \cos(x)\cos(y) - \sin(x)\sin(y)\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\sin(x)\sin(y) - \sin(y)\right) = \cos(x)\cos(y) - \cos(x)$$

$$\boxed{\frac{dy}{dx} = \frac{\cos(x)\left(\cos(y) - 1\right)}{\sin(y)\left(\sin(x) - 1\right)}}$$

§3.6 # 14 p. 243	$\frac{x^2}{9} + \frac{y^2}{36} = 1$ is an ellipse find tangent at $(-1, 4\sqrt{2})$

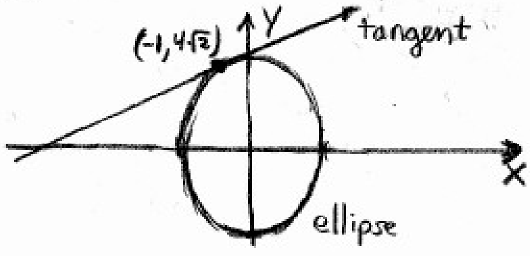
$\frac{d}{dx}\left(\frac{x^2}{9} + \frac{y^2}{36}\right) = \frac{d}{dx}(1) = 0$, Diff. implicitly to find $\frac{dy}{dx}$.

$$\frac{2}{9}x + \frac{2}{36}y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-\frac{2}{9}x}{\frac{2}{36}y} = \frac{-36x}{9y} = \frac{-4x}{y}$$

Then at $(-1, 4\sqrt{2})$ we have $\left.\frac{dy}{dx}\right|_{(-1,4\sqrt{2})} = \frac{-4(-1)}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = m$

Using point-slope form of line we get

$$\boxed{y = 4\sqrt{2} + \frac{1}{\sqrt{2}}(x+1)}$$



§3.6 #24 p. 244	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Show that the tangent thru (x_0, y_0) on the ellipse is $\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$

Differentiate the eq^n of ellipse to find,

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$$

Now at the point (x_0, y_0) we have slope

$$\left.\frac{dy}{dx}\right|_{(x_0,y_0)} = \frac{-b^2 x_0}{a^2 y_0}$$

Use point slope form to find line thru (x_0, y_0) with slope given by $\left.\frac{dy}{dx}\right|_{(x_0,y_0)}$ we get,

$$y = y_0 - \frac{b^2 x_0}{a^2 y_0}(x - x_0)$$

$$y = y_0 - \frac{b^2}{y_0}\frac{x_0 x}{a^2} + \frac{b^2 x_0^2}{a^2 y_0}$$ multiply by $\frac{y_0}{b^2}$ to see

$$\frac{y_0 y}{b^2} = \frac{y_0^2}{b^2} - \frac{x_0 x}{a^2} + \frac{x_0^2}{a^2}$$

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$$ because (x_0, y_0) is on the ellipse!

$$\therefore \boxed{\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1}$$

§3.6 #28 p. 244	Find derivative of function (explicitly in terms of x)

$$y = (\sin^{-1}(x))^2$$

$$\sqrt{y} = \sin^{-1}(x)$$

$$\sin(\sqrt{y}) = \sin(\sin^{-1}(x)) = x$$

- Alternatively you could use the derivative of $\sin^{-1}(x)$ and the chain-rule

Now differentiate to find $\frac{dy}{dx}$

$$1 = \cos(\sqrt{y})\frac{d}{dx}(\sqrt{y}) = \cos(\sqrt{y})\frac{1}{2\sqrt{y}}\frac{dy}{dx} \longrightarrow \frac{dy}{dx} = \frac{2\sqrt{y}}{\cos(\sqrt{y})}$$

We should give our answer in y, it should be in terms of x

$$\frac{dy}{dx} = \frac{2\sqrt{y}}{\cos(\sqrt{y})} = \boxed{\frac{2\sin^{-1}(x)}{\cos(\sin^{-1}(x))}}$$

§3.6 # 32 p. 244	find the derivative of the function $Y(x)$

$$Y = \tan^{-1}\left(x - \sqrt{1+x^2}\right)$$

$$\tan(Y) = x - \sqrt{1+x^2}$$

$$\sec^2(Y)\frac{dy}{dx} = 1 - \frac{1}{2\sqrt{1+x^2}}\frac{d}{dx}\left(1+x^2\right)$$

$$\frac{dy}{dx} = \frac{1 - \frac{x}{\sqrt{1+x^2}}}{\sec^2(Y)}$$

Notice $\sec^2(Y) = 1 + \tan^2(Y) = 1 + \left(x - \sqrt{1+x^2}\right)^2$. Thus $\frac{dy}{dx}$ becomes,

$$\boxed{\frac{dy}{dx} = \frac{1 - \frac{x}{\sqrt{1+x^2}}}{1 + \left(x - \sqrt{1+x^2}\right)^2}}$$

§ 3.6 #34 p. 244	The inverse for cosine is defined as the inverse for $\cos(x)$ for $0 \le x \le \pi$ where $\cos^{-1}(\cos(x)) = x$ & $\cos(\cos^{-1}(x)) = x$

derive the derivative of inverse cosine $Y = \cos^{-1}(x)$

$$Y = \cos^{-1}(x)$$

$$\cos(Y) = \cos(\cos^{-1}(x)) = x$$

$$-\sin(Y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{-1}{\sin(Y)} = \frac{-1}{\sqrt{1-\cos^2 Y}}$$

$\left(\text{since } \sin^2 Y = 1 - \cos^2 Y \Rightarrow \sin(Y) = \sqrt{1-\cos^2 Y}\right)$

Therefore $\boxed{\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}}$

Bonus Point: Solve #54 on pg. 245. Give me a neatly worked sol^{n}.

§3.7 # 2–12 p. 250	Differentiate remember $\frac{d}{dx}\ln(x) = \frac{1}{x}$

2.) $\frac{d}{dx}\left[\ln(x^2+10)\right] = \frac{1}{x^2+10}\frac{d}{dx}(x^2+10) = \boxed{\frac{2x}{x^2+10}}$

4.) $\frac{d}{dx}\left[\cos(\ln(x))\right] = -\sin(\ln(x))\frac{d}{dx}(\ln(x)) = \boxed{\frac{-\sin(\ln(x))}{x}}$

6.) $\frac{d}{dx}\left[\log_{10}\left(\frac{x}{x-1}\right)\right] = \frac{d}{dx}\left[\frac{1}{\ln(10)}\ln\left(\frac{x}{x-1}\right)\right]$

$= \frac{1}{\ln(10)}\frac{d}{dx}\left[\ln(x) - \ln(x-1)\right]$

$= \boxed{\frac{1}{\ln(10)}\left[\frac{1}{x} - \frac{1}{x-1}\right]}$

Change of base $\log_{10}(x) = \frac{\ln(x)}{\ln(10)}$

- question, why did I change to natural log.

8.) $\frac{d}{dx}\left(\ln\sqrt[5]{x}\right) = \frac{d}{dx}\left(\ln(x^{\frac{1}{5}})\right)$

$= \frac{d}{dx}\left(\frac{1}{5}\ln(x)\right)$

$= \boxed{\frac{1}{5x}}$

- Use the properties of Log to simplify expression <u>before</u> differentiating.

10.) $\frac{d}{dt}\left(\frac{1+\ln(t)}{1-\ln(t)}\right) = \frac{(1+\ln(t))'(1-\ln(t)) - (1+\ln(t))(1-\ln(t))'}{(1-\ln(t))^2}$

$= \frac{\frac{1}{t}(1-\cancel{\ln(t)}) - (1+\cancel{\ln(t)})\left(\frac{-1}{t}\right)}{(1-\ln(t))^2}$

$= \boxed{\frac{2}{t(1-\ln(t))^2}}$

12.) $\frac{d}{dx}\left[\ln(x+\sqrt{x^2-1})\right] = \frac{1}{x+\sqrt{x^2-1}}\frac{d}{dx}\left(x+\sqrt{x^2-1}\right)$

$= \frac{1}{x+\sqrt{x^2-1}}\left(1 + \frac{1}{2\sqrt{x^2-1}}\cdot\frac{d}{dx}(x^2-1)\right)$

$= \boxed{\frac{1}{x+\sqrt{x^2-1}}\left(1 + \frac{x}{\sqrt{x^2-1}}\right)}$

§ 3.7 #14-18 p. 250 — Differentiate, use log properties to simplify work.

14.) $\frac{d}{dx}\left[\ln(x^4\sin^2(x))\right] = \frac{d}{dx}\left[\ln(x^4) + \ln(\sin^2(x))\right]$

$= \frac{d}{dx}\left[4\ln(x) + 2\ln(\sin(x))\right]$

$= \frac{4}{x} + \frac{2}{\sin(x)}\frac{d}{dx}(\sin(x))$

$= \boxed{\frac{4}{x} + \frac{2\cos(x)}{\sin(x)}}$

16.) $\frac{d}{du}\left[\ln\sqrt{\frac{3u+2}{3u-2}}\right] = \frac{d}{du}\left[\frac{1}{2}\ln(3u+2) - \frac{1}{2}\ln(3u-2)\right]$

$= \frac{1}{2}\left[\frac{1}{3u+2}\frac{d}{du}(3u+2) - \frac{1}{3u-2}\frac{d}{du}(3u-2)\right]$

$= \boxed{\frac{3}{2}\left[\frac{1}{3u+2} - \frac{1}{3u-2}\right]}$

18.) $\frac{d}{dx}\left[(\ln(1+e^x))^2\right] = 2\ln(1+e^x)\frac{d}{dx}\left[\ln(1+e^x)\right]$

$= 2\ln(1+e^x)\frac{1}{1+e^x}\frac{d}{dx}(1+e^x)$

$= \boxed{2\ln(1+e^x)\frac{e^x}{1+e^x}}$

§ 3.7 # 28 p. 251 — Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = \sqrt{x}e^{x^2}(x^2+1)^{10}$

$\ln(y) = \ln\left(\sqrt{x}e^{x^2}(x^2+1)^{10}\right) = \frac{1}{2}\ln(x) + x^2 + 10\ln(x^2+1)$ ← why?

Now differentiate with respect to x implicitly,

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + 2x + \frac{10}{x^2+1}\frac{d}{dx}(x^2+1)$

Multiply by y and write it explicitly in terms of x,

$$\boxed{\frac{dy}{dx} = \sqrt{x}e^{x^2}(x^2+1)^{10}\left[\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right]}$$

§ 3.7 # 30–36 p. 251	Logarthmically differentriate

30.) $y = \sqrt[4]{\frac{x^2+1}{x^2-1}}$

$\ln(y) = \frac{1}{4}\ln\left(\frac{x^2+1}{x^2-1}\right) = \frac{1}{4}\left[\ln(x^2+1) - \ln(x^2-1)\right]$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{4}\left[\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right]$

$\boxed{\frac{dy}{dx} = \frac{1}{2}\sqrt[4]{\frac{x^2+1}{x^2-1}}\left(\frac{x}{x^2+1} - \frac{x}{x^2-1}\right)}$

32.) $y = x^{1/x}$

$\ln(y) = \ln(x^{\frac{1}{x}}) = \frac{1}{x}\ln(x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{-1}{x^2}\ln(x) + \frac{1}{x}\cdot\frac{1}{x}$

$\boxed{\frac{dy}{dx} = x^{1/x}\left(\frac{1-\ln(x)}{x^2}\right)}$

34.) $y = (\sin(x))^x$

$\ln(y) = \ln([\sin(x)]^x) = x\ln(\sin(x))$

$\frac{1}{y}\frac{dy}{dx} = \ln(\sin(x)) + x\frac{1}{\sin(x)}\cdot\cos(x)$

$\boxed{\frac{dy}{dx} = (\sin(x))^x\left[\ln(\sin(x)) + \frac{x\cos(x)}{\sin(x)}\right]}$

36.) $y = x^{\ln(x)}$

$\ln(y) = \ln(x^{\ln(x)}) = \ln(x)\cdot\ln(x) = (\ln(x))^2$

$\frac{1}{y}\frac{dy}{dx} = 2\ln(x)\cdot\frac{d}{dx}(\ln(x)) = \frac{2\ln(x)}{x}$

$\boxed{\frac{dy}{dx} = x^{\ln(x)}\left[\frac{2\ln(x)}{x}\right]}$