LINEAR ALGEBRA TOOLKIT

Transforming a matrix to reduced row echelon form

v. 1.23b

TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

Row Operation 1:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 6 & -1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

add **-1** times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -2 \\ 0 & 1 & 6 & -1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

Row Operation 2:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -2 \\ 0 & 1 & 6 & -1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

add **-2** times the 1st row to the 4th row

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -2 \\ 0 & 1 & 6 & -1 \\ 0 & -2 & -4 & -6 \end{bmatrix}$$

Row Operation 3:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -2 \\ 0 & 1 & 6 & -1 \\ 0 & -2 & -4 & -6 \end{bmatrix}$$

multiply the 2nd row by **-1/2**

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 1 & 6 & -1 \\ 0 & -2 & -4 & -6 \end{bmatrix}$$

Row Operation 4:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 1 & 6 & -1 \\ 0 & -2 & -4 & -6 \end{bmatrix}$$

add **-1** times the 2nd row to the 3rd row

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & \frac{9}{2} & -2 \\ 0 & -2 & -4 & -6 \end{bmatrix}$$

Row Operation 5:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & \frac{9}{2} & -2 \end{bmatrix}$$

add **2** times the 2nd row to the 4th row

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & \frac{9}{2} & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & -4 & -6 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -1 & -4 \end{bmatrix}$$

Row Operation 6:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & \frac{9}{2} & -2 \\ 0 & 0 & -1 & -4 \end{bmatrix}$$

multiply the 3rd row by **2/9**

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{-4}{9} \\ 0 & 0 & -1 & -4 \end{bmatrix}$$

Row Operation 7:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{-4}{9} \\ 0 & 0 & -1 & -4 \end{bmatrix}$$

add **1** times the 3rd row to the 4th row

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{-4}{9} \\ 0 & 0 & 0 & \frac{-40}{9} \end{bmatrix}$$

Row Operation 8:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{-4}{9} \\ 0 & 0 & 0 & \frac{-40}{9} \end{bmatrix}$$

multiply the 4th row by **-9/40**

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{-4}{9} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row Operation 9:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & \frac{-4}{9} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

add **4/9** times the 4th row to the 3rd row

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row Operation 10:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

add **-1** times the 4th row to the 2nd row

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row Operation 11:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

add **-4** times the 4th row to the 1st row

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row Operation 12:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

add **-3/2** times the 3rd row to the 2nd row

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row Operation 13:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

add **-3** times the 3rd row to the 1st row

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row Operation 14:

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

add **-2** times the 2nd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$