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P. Bogacki

# Transforming a matrix to reduced row echelon form

v. 1.23b

## TRANSFORMING MATRIX TO THE REDUCED ROW ECHELON FORM

Row  
Operation  
1:

$$\begin{bmatrix} 2 & 6 & -5 & 3 & 4 & 0 & 9 \\ 7 & 3 & 1 & 8 & 9 & 4 & 6 \\ 8 & -8 & 5 & 8 & 6 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

multiply the 1st row by  $\frac{1}{2}$

$$\begin{bmatrix} 1 & 3 & -\frac{5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 7 & 3 & 1 & 8 & 9 & 4 & 6 \\ 8 & -8 & 5 & 8 & 6 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Row  
Operation  
2:

$$\begin{bmatrix} 1 & 3 & -\frac{5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ & & 2 & 2 & & & 2 \\ 7 & 3 & 1 & 8 & 9 & 4 & 6 \\ 8 & -8 & 5 & 8 & 6 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

add -7 times the 1st row to the 2nd row

$$\begin{bmatrix} 1 & 3 & -\frac{5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ & & 2 & 2 & & & 2 \\ 0 & -18 & \frac{37}{2} & -\frac{5}{2} & -5 & 4 & -\frac{51}{2} \\ 8 & -8 & 5 & 8 & 6 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Row  
Operation  
3:

$$\begin{bmatrix} 1 & 3 & -\frac{5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ & & 2 & 2 & & & 2 \\ & & 37 & -5 & & & -51 \\ 0 & -18 & \frac{37}{2} & -\frac{5}{2} & -5 & 4 & -\frac{51}{2} \\ 8 & -8 & 5 & 8 & 6 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

add -8 times the 1st row to the 3rd row

$$\begin{bmatrix} 1 & 3 & -\frac{5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ & & 2 & 2 & & & 2 \\ & & 37 & -5 & & & -51 \\ 0 & -18 & \frac{37}{2} & -\frac{5}{2} & -5 & 4 & -\frac{51}{2} \\ 0 & -32 & 25 & -4 & -10 & 0 & -36 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Row  
Operation  
4:

$$\begin{bmatrix} 1 & 3 & -\frac{5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ & & 2 & 2 & & & 2 \\ & & 37 & -5 & & & -51 \\ 0 & -18 & \frac{37}{2} & -\frac{5}{2} & -5 & 4 & -\frac{51}{2} \\ 0 & -32 & 25 & -4 & -10 & 0 & -36 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

add -1 times the 1st row to the 4th row

$$\begin{bmatrix} 1 & 3 & -\frac{5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ & & 2 & 2 & & & 2 \\ & & 37 & -5 & & & -51 \\ 0 & -18 & \frac{37}{2} & -\frac{5}{2} & -5 & 4 & -\frac{51}{2} \\ 0 & -32 & 25 & -4 & -10 & 0 & -36 \\ 0 & -2 & \frac{7}{2} & -\frac{1}{2} & -1 & 1 & -\frac{7}{2} \end{bmatrix}$$

$$\begin{array}{ccc} 2 & 2 & 2 \end{array}$$

Row Operation 5:

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 0 & -18 & \frac{37}{2} & \frac{-5}{2} & -5 & 4 & \frac{-51}{2} \\ 0 & -32 & 25 & -4 & -10 & 0 & -36 \\ 0 & -2 & \frac{7}{2} & \frac{-1}{2} & -1 & 1 & \frac{-7}{2} \end{array}$$

multiply the 2nd row by  $-1/18$

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 0 & 1 & \frac{-37}{18} & \frac{5}{18} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{18} \\ 0 & -32 & 25 & -4 & -10 & 0 & -36 \\ 0 & -2 & \frac{7}{2} & \frac{-1}{2} & -1 & 1 & \frac{-7}{2} \end{array}$$

Row Operation 6:

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 0 & 1 & \frac{-37}{18} & \frac{5}{18} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{18} \\ 0 & -32 & 25 & -4 & -10 & 0 & -36 \\ 0 & -2 & \frac{7}{2} & \frac{-1}{2} & -1 & 1 & \frac{-7}{2} \end{array}$$

add 32 times the 2nd row to the 3rd row

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 0 & 1 & \frac{-37}{18} & \frac{5}{18} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{18} \\ 0 & 0 & \frac{-71}{18} & \frac{4}{9} & \frac{-10}{9} & \frac{-64}{9} & \frac{28}{9} \\ 0 & -2 & \frac{7}{2} & \frac{-1}{2} & -1 & 1 & \frac{-7}{2} \end{array}$$

Row Operation 7:

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 0 & 1 & \frac{-37}{18} & \frac{5}{18} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{18} \\ 0 & 0 & \frac{-71}{18} & \frac{4}{9} & \frac{-10}{9} & \frac{-64}{9} & \frac{28}{9} \\ 0 & -2 & \frac{7}{2} & \frac{-1}{2} & -1 & 1 & \frac{-7}{2} \end{array}$$

add 2 times the 2nd row to the 4th row

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 0 & 1 & \frac{-37}{18} & \frac{5}{18} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{18} \\ 0 & 0 & \frac{-71}{18} & \frac{4}{9} & \frac{-10}{9} & \frac{-64}{9} & \frac{28}{9} \\ 0 & 0 & \frac{13}{9} & \frac{-2}{9} & \frac{-4}{9} & \frac{5}{9} & \frac{-2}{9} \end{array}$$

Row Operation 8:

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \\ 0 & 0 & \frac{13}{9} & \frac{-2}{9} & \frac{-4}{9} & \frac{5}{9} & \frac{-2}{9} \end{array}$$

multiply the 3rd row by  $-9/71$

$$\begin{array}{cccccc|c} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} \end{array}$$

$$\begin{array}{cccc|cccc} 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline 0 & 0 & \frac{-71}{9} & \frac{4}{9} & \frac{-10}{9} & \frac{-64}{9} & \frac{28}{3} & \\ \hline 0 & 0 & \frac{13}{9} & \frac{-2}{9} & \frac{-4}{9} & \frac{5}{9} & \frac{-2}{3} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

$$\begin{array}{cccc|cccc} 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline 0 & 0 & 1 & \frac{-4}{71} & \frac{10}{71} & \frac{64}{71} & \frac{-84}{71} & \\ \hline 0 & 0 & \frac{13}{9} & \frac{-2}{9} & \frac{-4}{9} & \frac{5}{9} & \frac{-2}{3} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

Row Operation 9:

$$\begin{array}{cccc|cccc} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} & \\ \hline 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline 0 & 0 & 1 & \frac{-4}{71} & \frac{10}{71} & \frac{64}{71} & \frac{-84}{71} & \\ \hline 0 & 0 & \frac{13}{9} & \frac{-2}{9} & \frac{-4}{9} & \frac{5}{9} & \frac{-2}{3} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

add  $-\frac{13}{9}$  times the 3rd row to the 4th row

$$\begin{array}{cccc|cccc} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} & \\ \hline 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline 0 & 0 & 1 & \frac{-4}{71} & \frac{10}{71} & \frac{64}{71} & \frac{-84}{71} & \\ \hline 0 & 0 & 0 & \frac{-10}{71} & \frac{-46}{71} & \frac{-53}{71} & \frac{74}{71} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

Row Operation 10:

$$\begin{array}{cccc|cccc} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} & \\ \hline 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline 0 & 0 & 1 & \frac{-4}{71} & \frac{10}{71} & \frac{64}{71} & \frac{-84}{71} & \\ \hline 0 & 0 & 0 & \frac{-10}{71} & \frac{-46}{71} & \frac{-53}{71} & \frac{74}{71} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

multiply the 4th row by  $-\frac{71}{10}$

$$\begin{array}{cccc|cccc} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} & \\ \hline 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline 0 & 0 & 1 & \frac{-4}{71} & \frac{10}{71} & \frac{64}{71} & \frac{-84}{71} & \\ \hline 0 & 0 & 0 & 1 & \frac{23}{5} & \frac{53}{10} & \frac{-37}{5} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

Row Operation 11:

$$\begin{array}{cccc|cccc} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} & \\ \hline 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

add  $\frac{4}{71}$  times the 4th row to the 3rd row

$$\begin{array}{cccc|cccc} 1 & 3 & \frac{-5}{2} & \frac{3}{2} & 2 & 0 & \frac{9}{2} & \\ \hline 0 & 1 & \frac{-37}{36} & \frac{5}{36} & \frac{5}{18} & \frac{-2}{9} & \frac{17}{12} & \\ \hline & & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{9}{9} & \frac{3}{3} & \end{array}$$

$$\begin{array}{cccc|ccc}
 0 & 0 & 1 & & -4 & 10 & 64 & -84 \\
 & & & & 71 & 71 & 71 & 71 \\
 & & & & & 23 & 53 & -37 \\
 0 & 0 & 0 & 1 & & & & \\
 & & & & 5 & 10 & & 5
 \end{array}$$

$$\begin{array}{cccc|ccc}
 0 & 0 & 1 & 0 & & 2 & 6 & -8 \\
 & & & & & 5 & 5 & 5 \\
 & & & & & 23 & 53 & -37 \\
 0 & 0 & 0 & 1 & & & & \\
 & & & & 5 & 10 & & 5
 \end{array}$$

Row Operation 12:

$$\begin{array}{cccc|ccc}
 1 & 3 & & & -5 & 3 & & 9 \\
 & & & & 2 & 2 & & 2 \\
 0 & 1 & & & -37 & 5 & 5 & -2 & 17 \\
 & & & & 36 & 36 & 18 & 9 & 12 \\
 0 & 0 & 1 & 0 & & 2 & 6 & -8 \\
 & & & & & 5 & 5 & 5 \\
 & & & & & 23 & 53 & -37 \\
 0 & 0 & 0 & 1 & & 5 & 10 & 5
 \end{array}$$

add  $-5/36$  times the 4th row to the 2nd row

$$\begin{array}{cccc|ccc}
 1 & 3 & & & -5 & 3 & & 9 \\
 & & & & 2 & 2 & & 2 \\
 0 & 1 & & & -37 & 0 & -13 & -23 & 22 \\
 & & & & 36 & 36 & 24 & 9 & \\
 0 & 0 & 1 & 0 & & 2 & 6 & -8 \\
 & & & & & 5 & 5 & 5 \\
 & & & & & 23 & 53 & -37 \\
 0 & 0 & 0 & 1 & & 5 & 10 & 5
 \end{array}$$

Row Operation 13:

$$\begin{array}{cccc|ccc}
 1 & 3 & & & -5 & 3 & & 9 \\
 & & & & 2 & 2 & & 2 \\
 0 & 1 & & & -37 & & -13 & -23 & 22 \\
 & & & & 36 & & 36 & 24 & 9 \\
 0 & 0 & 1 & 0 & & 2 & 6 & -8 \\
 & & & & & 5 & 5 & 5 \\
 & & & & & 23 & 53 & -37 \\
 0 & 0 & 0 & 1 & & 5 & 10 & 5
 \end{array}$$

add  $-3/2$  times the 4th row to the 1st row

$$\begin{array}{cccc|ccc}
 1 & 3 & & & -5 & 0 & -49 & -159 & 78 \\
 & & & & 2 & & 10 & 20 & 5 \\
 0 & 1 & & & -37 & & -13 & -23 & 22 \\
 & & & & 36 & & 36 & 24 & 9 \\
 0 & 0 & 1 & 0 & & 2 & 6 & -8 \\
 & & & & & 5 & 5 & 5 \\
 & & & & & 23 & 53 & -37 \\
 0 & 0 & 0 & 1 & & 5 & 10 & 5
 \end{array}$$

Row Operation 14:

$$\begin{array}{cccc|ccc}
 1 & 3 & & & -5 & & -49 & -159 & 78 \\
 & & & & 2 & & 10 & 20 & 5 \\
 0 & 1 & & & -37 & & -13 & -23 & 22 \\
 & & & & 36 & & 36 & 24 & 9 \\
 0 & 0 & 1 & 0 & & 2 & 6 & -8 \\
 & & & & & 5 & 5 & 5 & 
 \end{array}$$

add  $37/36$  times the 3rd row to the 2nd row

$$\begin{array}{cccc|ccc}
 1 & 3 & & & -5 & & -49 & -159 & 78 \\
 & & & & 2 & & 10 & 20 & 5 \\
 0 & 1 & 0 & 0 & & 1 & 11 & 4 & \\
 & & & & & 20 & 40 & 5 & \\
 0 & 0 & 1 & 0 & & 2 & 6 & -8 \\
 & & & & & 5 & 5 & 5 & 
 \end{array}$$

$$\left[ \begin{array}{cccc|ccc} 0 & 0 & 0 & 1 & \frac{23}{5} & \frac{53}{10} & \frac{-37}{5} \end{array} \right]$$

$$\left[ \begin{array}{cccc|ccc} 0 & 0 & 0 & 1 & \frac{23}{5} & \frac{53}{10} & \frac{-37}{5} \end{array} \right]$$

Row  
Operation  
15:

$$\left[ \begin{array}{cccc|ccc} 1 & 3 & \frac{-5}{2} & 0 & \frac{-49}{10} & \frac{-159}{20} & \frac{78}{5} \\ 0 & 1 & 0 & 0 & 1 & 11 & 4 \\ 0 & 0 & 1 & 0 & \frac{2}{5} & \frac{6}{5} & \frac{-8}{5} \\ 0 & 0 & 0 & 1 & \frac{23}{5} & \frac{53}{10} & \frac{-37}{5} \end{array} \right]$$

add  $\frac{5}{2}$  times the 3rd  
row to the 1st row

$$\left[ \begin{array}{cccc|ccc} 1 & 3 & 0 & 0 & \frac{-39}{10} & \frac{-99}{20} & \frac{58}{5} \\ 0 & 1 & 0 & 0 & 1 & 11 & 4 \\ 0 & 0 & 1 & 0 & \frac{2}{5} & \frac{6}{5} & \frac{-8}{5} \\ 0 & 0 & 0 & 1 & \frac{23}{5} & \frac{53}{10} & \frac{-37}{5} \end{array} \right]$$

Row  
Operation  
16:

$$\left[ \begin{array}{cccc|ccc} 1 & 3 & 0 & 0 & \frac{-39}{10} & \frac{-99}{20} & \frac{58}{5} \\ 0 & 1 & 0 & 0 & 1 & 11 & 4 \\ 0 & 0 & 1 & 0 & \frac{2}{5} & \frac{6}{5} & \frac{-8}{5} \\ 0 & 0 & 0 & 1 & \frac{23}{5} & \frac{53}{10} & \frac{-37}{5} \end{array} \right]$$

add  $-3$  times the 2nd  
row to the 1st row

$$\left[ \begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & \frac{-81}{10} & \frac{-231}{20} & \frac{46}{5} \\ 0 & 1 & 0 & 0 & 1 & 11 & 4 \\ 0 & 0 & 1 & 0 & \frac{2}{5} & \frac{6}{5} & \frac{-8}{5} \\ 0 & 0 & 0 & 1 & \frac{23}{5} & \frac{53}{10} & \frac{-37}{5} \end{array} \right]$$