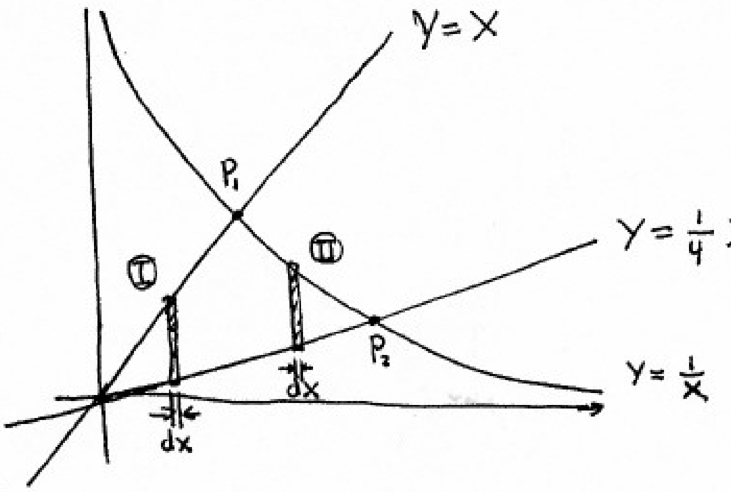
§6.1 # 15 : Area bounded by $y = 1/x$, $y = x$, $y = \frac{1}{4}x$, $x > 0$



we will need to break this up into 2 cases I'll call them Ⅰ & Ⅱ

We should determine the points of intersection, call them P_1 & P_2

P_1 has $\frac{1}{x} = x \Rightarrow 1 = x^2 \Rightarrow x = \pm 1$ but $x > 0 \therefore \underline{x = 1}$

$\therefore \underline{P_1 = (1,1)}$

P_2 has $\frac{1}{x} = \frac{1}{4}x \Rightarrow 4 = x^2 \Rightarrow x = \pm 2$ but $x > 0 \therefore x = 2$

$\therefore \underline{P_2 = (2, 1/2)}$

Now we can clearly see that Ⅰ has $0 \leq x \leq 1$ whereas Ⅱ lies somewhere in $1 \leq x \leq 2$. Notice then

Ⅰ has $h = x - \frac{1}{4}x \Rightarrow dA = (x - \frac{1}{4}x)dx = \frac{3}{4}x\,dx$

Ⅱ has $h = \frac{1}{x} - \frac{1}{4}x \Rightarrow dA = (\frac{1}{x} - \frac{1}{4}x)dx$

Now we add-up the area of all the tiny rectangles by integrating,

$$A = \int_0^1 \frac{3}{4}x\,dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right)dx$$

$$= \frac{3}{8}x^2\Big|_0^1 + \left(\ln|x| - \frac{1}{8}x^2\right|_1^2$$

$$= \frac{3}{8}(1-0) + \left(\ln(2) - \frac{1}{8}(4)\right) - \left(\underbrace{\ln(1)}_{=0} - \frac{1}{8}\right)$$

$$= 3/8 + \ln(2) - 1/2 + 1/8$$

$$= \boxed{\ln(2)}$$