Homework 30, Calculus I

§5.5#1

$$\int \cos(3x)\,dx = \int \cos(u)\,\frac{du}{3} \quad \leftarrow \boxed{\begin{array}{l} u = 3x \\ du = 3dx \end{array}}$$

$$= \frac{1}{3}\cos(u) + C$$

$$= \boxed{\frac{1}{3}\cos(3x) + C}$$

§5.5#5

$$\int \cos^3\theta \sin\theta\, d\theta = \int u^3(-du) \quad \leftarrow \boxed{\begin{array}{l} u = \cos\theta \\ du = -\sin\theta\, d\theta \\ \sin\theta\, d\theta = -du \end{array}}$$

$$= \frac{-1}{4}u^4 + C$$

$$= \boxed{\frac{-1}{4}\cos^4\theta + C}$$

§5.5#7

$$\int x\sin(x^2)dx = \int \frac{1}{2}\sin(u)\,du \quad \leftarrow \boxed{\begin{array}{l} u = x^2 \\ du = 2x\,dx \\ x\,dx = \frac{1}{2}du \end{array}}$$

$$= \frac{-1}{2}\cos(u) + C$$

$$= \boxed{\frac{-1}{2}\cos(x^2) + C}$$

§5.5#9

$$\int (3x-2)^{20}dx = \int \frac{1}{3}u^{20}\,du \quad \leftarrow \boxed{\begin{array}{l} u = 3x-2 \\ du = 3dx \\ dx = \frac{1}{3}du \end{array}}$$

$$= \frac{1}{3(21)}u^{21} + C$$

$$= \boxed{\frac{1}{63}(3x-2)^{21} + C}$$

§5.5#11

$$\int (x+1)\sqrt{2x+x^2}\,dx = \int \frac{1}{2}\sqrt{u}\,du$$

$$= \frac{1}{3}u^{3/2} + C$$

$$= \boxed{\frac{1}{3}(2x+x^2)^{3/2} + C}$$

$$\boxed{\begin{array}{l} u = 2x + x^2 \\ du = (2+2x)dx \\ (x+1)dx = \frac{1}{2}du \end{array}}$$

§5.5#13

$$\int \sin(\pi t)\,dt = \int \sin(u)\,\frac{1}{\pi}\,du = \boxed{\frac{-1}{\pi}\cos(\pi t) + C}$$

$u = \pi t$, $du = \pi\,dt$, $dt = \frac{1}{\pi}\,du$

§5.5#15

$$\int \frac{a + bx^2}{\sqrt{3ax + bx^3}}\,dx = \int \frac{1}{3}\,u^{-1/2}\,du = \boxed{\frac{2}{3}\sqrt{3ax + bx^3} + C}$$

$u = 3ax + bx^3$, $du = (3a + 3bx^2)\,dx$, $\frac{1}{3}\,du = (a + bx^2)\,dx$

§5.5#17

$$\int \frac{\cos(-\sqrt{t})}{\sqrt{t}}\,dt = \int 2\cos(-\sqrt{t})\,d(-\sqrt{t}) = \boxed{2\sin(\sqrt{t}) + C}$$

$\frac{d(\sqrt{t})}{dt} = \frac{1}{2\sqrt{t}}$, $2\,d(\sqrt{t}) = \frac{dt}{\sqrt{t}}$

§5.5#19

$$\int \cos\theta \sin^6\theta\,d\theta = \int \sin^6\theta\,d(\sin\theta) = \boxed{\frac{1}{7}\sin^7\theta + C}$$

$d(\sin\theta) = \cos\theta\,d\theta$

Remark: if you like this notation feel free to use it.

§5.5#21

$$\int \frac{z^2}{\sqrt[3]{1 + z^3}}\,dz = \int u^{-1/3}\,\frac{1}{3}\,du = \frac{1}{3}\left(\frac{3}{2}\,u^{2/3}\right) + C = \boxed{\frac{1}{2}\left(1 + z^3\right)^{2/3} + C}$$

$u = 1 + z^3$, $du = 3z^2\,dz$

§5.5#23

$$\int \sqrt{\cot(x)}\,\csc^2(x)\,dx = \int \sqrt{u}\,(-du) = -\frac{2}{3}\,u^{3/2} + C = \boxed{\frac{-2}{3}\left(\cot(x)\right)^{3/2} + C}$$

$u = \cot(x)$, $-du = \csc^2(x)\,dx$

§5.5 # 25

$$\int \sec^3(x)\tan(x)\,dx = \int \sec^2(x)\sec(x)\tan(x)\,dx$$

$$= \int u^2\,du \quad \leftarrow \boxed{\begin{array}{l} u = \sec(x) \\ du = \sec(x)\tan(x)\,dx \end{array}}$$

$$= \frac{1}{3}u^3 + C$$

$$= \boxed{\frac{1}{3}\sec^3(x) + C}$$

§5.5 # 27

$$\int \frac{\cos(x)\,dx}{\sin^2(x)} = \int \frac{du}{u^2} \quad \leftarrow \boxed{\begin{array}{l} u = \sin(x) \\ du = \cos(x)\,dx \end{array}}$$

$$= \frac{-1}{u} + C$$

$$= \boxed{\frac{-1}{\sin(x)} + C}$$

§5.5 # 29

$$\int \frac{x}{\sqrt[4]{x+2}}\,dx = \int \frac{u-2}{\sqrt[4]{u}}\,du \quad \leftarrow \boxed{\begin{array}{l} u = x+2 \\ du = dx \\ x = u-2 \end{array}}$$

$$= \int \left(u^{3/4} - 2u^{-1/4}\right)du$$

$$= \frac{4}{7}u^{7/4} - 2\left(\frac{4}{3}\right)u^{3/4} + C$$

$$= \boxed{\frac{4}{7}(x+2)^{7/4} - \frac{8}{3}(x+2)^{3/4} + C}$$