Homework 31, Calculus I

§5.5 # 35

$$\int_0^2 (x-1)^{25}\,dx = \frac{1}{26}u^{26}\Big|_{-1}^{1}$$

$$= \frac{1}{26}(1-1)$$

$$= \boxed{0}.$$

$u = x-1$
$du = dx$
$u(2) = 2-1 = 1$
$u(0) = 0-1 = -1$

§5.5 # 37

$$\int_0^1 x^2(1+2x^3)^5\,dx = \int_1^3 \frac{1}{6}u^5\,du$$

$$= \frac{1}{36}u^6\Big|_1^3$$

$$= \frac{1}{36}(3^6-1)$$

$$= \frac{728}{36}$$

$$= \boxed{\frac{182}{9}}$$

$u = 1+2x^3$
$du = 6x^2\,dx$
$u(0) = 1+2(0) = 1$
$u(1) = 3.$

§5.5 # 39

$$\int_0^{\pi} \sec^2(t/4)\,dt = 4\tan(t/4)\Big|_0^{\pi}$$

$$= 4\tan(\pi/4) - 4\tan(0)$$

$$= \boxed{4}$$

§5.5 # 41

$$\int_{-\pi/6}^{\pi/6} \tan^3\theta\,d\theta = \int_{-\pi/6}^{\pi/6} (\sec^2\theta - 1)\tan\theta\,d\theta$$

$$= \int_{-\pi/6}^{\pi/6} \sec\theta\sec\theta\tan\theta\,d\theta - \int_{-\pi/6}^{\pi/6} \tan\theta\,d\theta$$

$$= \int_{-2/\sqrt{3}}^{2/\sqrt{3}} u\,du - \ln|\sec\theta|\Big|_{-\pi/6}^{\pi/6}$$

$$= \frac{1}{2}\left(\left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{-2}{\sqrt{3}}\right)^2\right) - \ln\left|\frac{2}{\sqrt{3}}\right| + \ln\left|\frac{2}{\sqrt{3}}\right| = \boxed{0}.$$

$u = \sec\theta$
$du = \sec\theta\tan\theta\,d\theta$
$u(\pi/6) = \sec(\pi/6)$
$= \frac{1}{\cos(\pi/6)}$
$= \frac{2}{\sqrt{3}}$

§5.5# 43

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^{27} \frac{1}{2} u^{-2/3}\, du$$

$u = 1+2x \quad u(13) = 27$
$du = 2dx \quad u(0) = 1$
$dx = \frac{1}{2}du$

$$= \frac{3}{2} u^{1/3} \Big|_1^{27}$$

$$= \frac{3}{2}\left(\sqrt[3]{27} - \sqrt[3]{1}\right) \qquad \sqrt[3]{27} = 3.$$

$$= \boxed{3}$$

§5.5# 45

$$\int_0^a x\sqrt{x^2+a^2}\, dx = \int_{a^2}^{2a^2} \frac{1}{2} u^{1/2}\, du$$

$u = x^2+a^2 \quad u(a) = 2a^2$
$du = 2x\,dx \quad u(0) = a^2$
$x\,dx = \frac{1}{2}du$

$$= \frac{1}{3} u^{3/2} \Big|_{a^2}^{2a^2}$$

$$= \frac{1}{3}(2a^2)^{3/2} - \frac{1}{3}(a^2)^{3/2}$$

$$= \boxed{\frac{1}{3}\left(2^{3/2} - 1\right) a^3}$$

§5.5# 47

$$\int_1^2 x\sqrt{x-1}\, dx = \int_0^1 (u+1)\sqrt{u}\, du$$

$u = x-1 \quad u(2) = 1$
$du = dx \quad u(1) = 0$
$x = u+1$

$$= \int_0^1 \left(u^{3/2} + u^{1/2}\right) du$$

$$= \frac{2}{5} + \frac{2}{3}$$

$$= \boxed{16/15}$$

§5.5# 49

$$\int_{1/2}^1 \cos(x^{-2})\frac{dx}{x^3} = \int_4^1 \frac{-1}{2}\cos(u)\, du$$

$u = x^{-2} \quad u(1) = 1$
$du = \frac{-2}{x^3}dx \quad u(1/2) = \left(\frac{1}{2}\right)^{-2} = 4.$
$-\frac{1}{2}du = \frac{dx}{x^3}$

$$= -\frac{1}{2}\sin(u)\Big|_4^1$$

$$= \boxed{\frac{1}{2}\left(\sin(4) - \sin(1)\right)}$$