Homework 35, Calculus I

§7.6 # 63 See §5.5 # 73 Homework #32 Sol'n

§7.6 # 65

$$\int \frac{1}{\sin^{-1}(x)} \frac{dx}{\sqrt{1-x^2}} = \int \frac{du}{u} \quad \leftarrow \quad u = \sin^{-1}(x),\ du = \frac{dx}{\sqrt{1-x^2}}$$

$$= \ln|u| + C$$

$$= \boxed{\ln|\sin^{-1}(x)| + C}$$

§7.6 # 69

$$\int \frac{1}{1+x} \frac{dx}{\sqrt{x}} = \int \frac{2du}{1+u^2} \quad \leftarrow \quad u = \sqrt{x},\ 1+x = 1+u^2,\ du = \frac{1}{2\sqrt{x}}dx,\ \frac{dx}{\sqrt{x}} = 2du$$

$$= 2\tan^{-1}(u) + C$$

$$= \boxed{2\tan^{-1}(\sqrt{x}) + C}$$

§7.6 # 71

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{dx}{\sqrt{a^2(1 - x^2/a^2}} \quad \rightarrow (*)$$

$$= \frac{1}{a}\int \frac{dx}{\sqrt{1 - x^2/a^2}} \quad \leftarrow (*)$$

$$= \frac{1}{a}\int \frac{a\,du}{\sqrt{1-u^2}} \qquad u = x/a,\ du = \frac{1}{a}dx,\ a\,du = dx$$

$$= \int \frac{du}{\sqrt{1-u^2}}$$

$$= \sin^{-1}(u) + C$$

$$= \boxed{\sin^{-1}(x/a) + C}$$

(*) ($a > 0$ is needed to conclude $\sqrt{a^2} = a$).