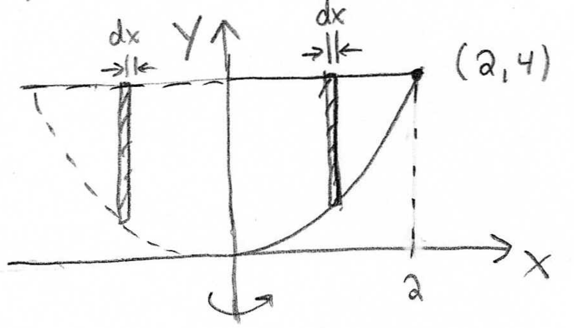
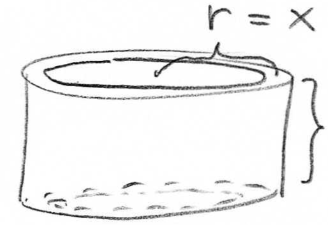
Homework 39, Calculus I

§6.3 #5 Use method of cylindrical shells to find volume of solid generated by rotation of area bounded by $Y = x^2$, $0 \le x \le 2$, $y = 4$ and $x = 0$ around y-axis.





$$dV = 2\pi r h\, dx = 2\pi x(4-x^2)dx$$

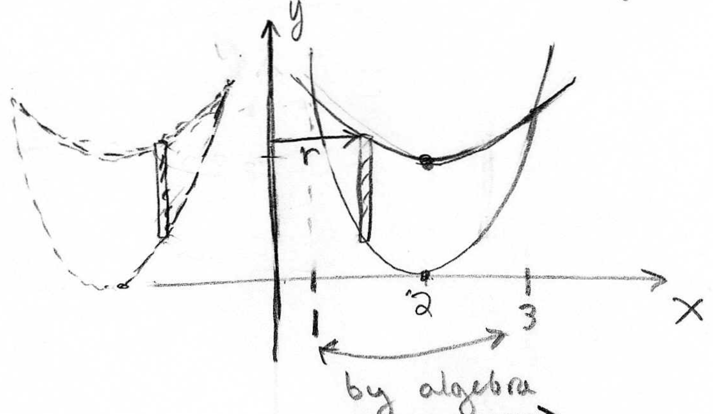
for each x with $0 \le x \le 2$.

$$V = \int_0^2 2\pi(4x - x^3)\,dx = 2\pi\left(2x^2 - \tfrac{1}{4}x^4\right)\Big|_0^2 = 2\pi(8-4) = \boxed{8\pi}$$

---

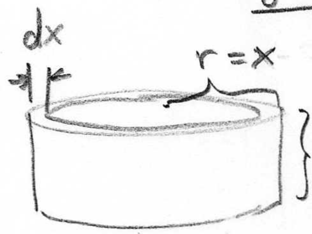
§6.3 #7 $\underbrace{Y = 4(x-2)^2}_{\text{vertex } (2,0)}$ and $\underbrace{y = x^2 - 4x + 7}_{y = (x-2)^2 + 3}$ around y-axis.

vertex at $(2,3)$.



Intersection of parabolas at

$$4(x-2)^2 = x^2 - 4x + 7$$
$$4(x^2 - 4x + 4) = x^2 - 4x + 7$$
$$4x^2 - 16x + 16 = x^2 - 4x + 7$$
$$3x^2 - 12x + 9 = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-1)(x-3) = 0$$
$$x = 1,\ x = 3$$

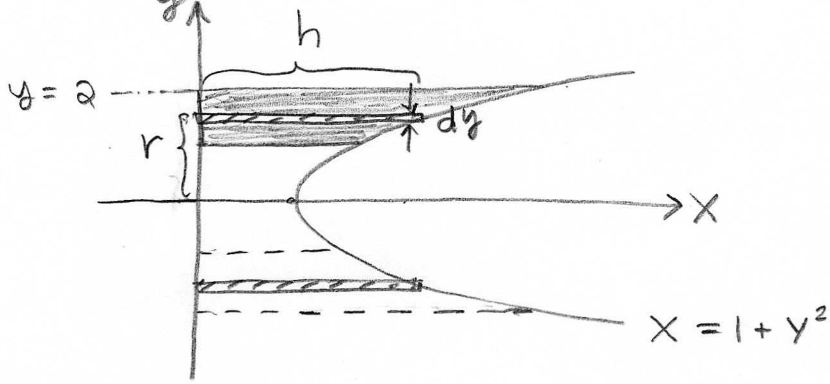


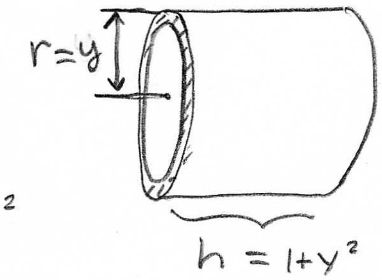
$$h = (x^2 - 4x + 7) - 4(x-2)^2 = -3x^2 + 12x - 9.$$

$$dV = 2\pi r h\, dx = 6\pi x(4x - x^2 - 3)\,dx$$

$$V = \int_1^3 6\pi(4x^2 - x^3 - 3x)\,dx$$
$$= 6\pi\left(\tfrac{4}{3}(27) - \tfrac{1}{4}(81) - \tfrac{3}{2}(9) - \tfrac{4}{3} + \tfrac{1}{4} + \tfrac{3}{2}\right)$$
$$= \boxed{16\pi}$$

§6.3 #9 Find volume of solid obtained from rotating area bounded by $x = 1 + y^2$, $x = 0$, $y = 1$, $y = 2$ around x-axis,





$$dV = (2\pi r h)\,dy = 2\pi y(1+y^2)\,dy$$

$$V = \int_1^2 \pi(2y + 2y^3)\,dy = \pi\left(y^2\Big|_1^2 + \frac{1}{2} y^4\Big|_1^2\right)$$

$$= \pi\left(4 - 1 + \tfrac{1}{2}(16-1)\right)$$

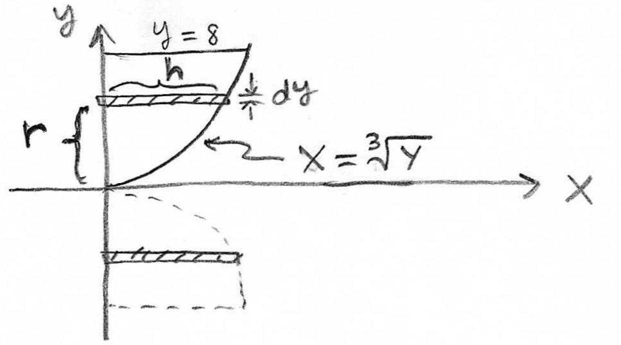
$$= \pi\left(3 + \tfrac{15}{2}\right)$$

$$= \boxed{21\pi/2}$$

§6.3 #11 $y = x^3$, $y = 8$, $x = 0$ same as #9,

$r = y$

$h = \sqrt[3]{y}$



$$dV = (2\pi r h)\,dy$$

$$= 2\pi y \sqrt[3]{y}\,dy$$

$$= 2\pi y^{4/3}\,dy$$

for each $y \in [0, 8]$.

$$V = \int_0^8 2\pi y^{4/3}\,dy$$

$$= \frac{6}{7}\pi y^{7/3}\Big|_0^8$$

$$= \frac{6\pi}{7}\left(8^{7/3}\right)$$

$$= \frac{6\pi}{7}\left(2^7\right)$$

$$= \boxed{\frac{768\pi}{7}}$$

§6.3 #46 See E15 in my notes. (Example 7.4.15)