

If the growth of a population P is proportional to its size then

$$\frac{dP}{dt} = kP$$

Likewise if the rate of change of Y is proportional to Y

$$\frac{dY}{dt} = kY$$

As discussed in [E1] of § 7.3

$$Y(t) = Y_0 e^{kt} \quad \text{where } Y_0 = Y(0).$$

Like wise $P(t) = P_0 e^{kt}$ where $P_0 = P(0)$. Both follow simply from sep. of variables. By the way:

$$\frac{dP}{dt} = kP \Leftrightarrow \frac{1}{P} \frac{dP}{dt} = k \Leftrightarrow \text{relative growth rate constant}$$

So $k \equiv$ the relative growth rate.

[E1] If the population doubles every 10 yrs what is k ?

$$P(0) = P_0 \quad \text{and} \quad P(10) = 2P_0 = P_0 e^{10k}$$

$$\Rightarrow \ln(2) = 10k \Rightarrow k = \frac{\ln(2)}{10 \text{ yr.}} = 0.0693 \frac{1}{\text{yr.}}$$

The relative growth rate is 6.93%.

[E2] Let $Y(t) = m(t)$ be the mass of some radioactive substance then as the mass destabilizes via radiation we have

$$\frac{dm}{dt} = km \quad (k < 0 \text{ since } m \text{ is decreasing})$$

$$\Rightarrow m(t) = m_0 e^{kt}$$

If balonium has a half-life of 1 yr. then what percentage of the balonium is still in the fridge after $\frac{1}{10}$ yr.?

$$m(1) = \frac{m_0}{2} = m_0 e^k \quad \therefore k = \ln(1/2) = -0.693 = k$$

$$\therefore m\left(\frac{1}{10}\right) = m_0 e^{-0.693\left(\frac{1}{10}\right)} = (0.933)m_0 \Rightarrow \boxed{93.3\% \text{ remains}}$$