Introduction to Diff Eq^{n}'s

A diff. eq^{n} is an eq^{n} involving derivatives. An ordinary diff eq^{n} (or ODE) involves a single independent variable (say x) and a dependent variable along with it's derivatives (say $y, y', y'', \ldots y^{(n)}$). We define the order of the ODE to be the highest derivative in it. If we can set $y^{(n)} = f(y, y', .., y^{(n-1)})$ ← no x then the ODE is autonomous the independent variable x does not explicitly appear. $w(y^{(n)}, y^{(n-1)}, .., y'', y', y) = 0$ then it homogeneous

$$y'' - 3y' = 5x \qquad (2^{nd} \text{ degree, non-homogeneous})$$

$$y'' - 3y' = 0 \qquad (2^{nd} \text{ deg., homogeneous})$$

$$y' = y^2 \qquad (1^{st} \text{ deg, homogeneous})$$

$$y''' - 3y' = 5 \qquad (3^{rd} \text{ deg, non-homogeneous})$$

<u>Remark</u>: this is a generlization of what we've been doing

$$\frac{dy}{dx} = w(x), \quad \text{find } f(x,c) \text{ so that } \frac{d}{dx}(f) = w$$

is the same as integrating. $\frac{d}{dx}(f) = \cos(x) \Rightarrow f(x,c) = \sin(x) + c$.
So we can already solve many 1^{st} order ode's, but not all are even recognizable as integrals (it takes work to rewrite them as such)

<u>FACT</u>: A n-degree ODEq^{n} will have n-arbitrary constants in it's general sol^{n}. In order to pick out a particular sol^{n} we must give some extra boundary conditions or initial conditions.

> Def^{n}/ If $F(y^{(n)}, y^{(n-1)}, .., y''', y'', y', x) = 0$ then $f(x)$ is a sol^{n} to the differential eq^{n} if $F(y^{(n)}, y^{(n-1)}, .., x) = 0$ when $y = f(x)$.
> In simple terms if you substitute that fnct. into the ODE it satisfies it!

$\boxed{E1}$ $y'' = -y$ has sol^{n} $y = \cos t$ since $y' = -\sin t$, $y'' = -\cos t$

$\therefore y'' = -\cos t = -y$ when $y = \cos t$. //