Probability

Let X be a continous random variable then we can describe the probability of a certain range of X occuring via an integral

$$P[a \le X \le b] = \int_a^b f(x)\,dx$$

The function $f(x)$ is the probability density function. In particular for this to make sense the sum of all prob. should be one

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \text{Normalization Condition}}$$

$\boxed{E1}$ Let $f(x) = Ae^{-x/\tau}$ for $x \in D = [0, \infty)$ then what value must we assign to A to make $f(x)$ a prob. density and what is $P[0 \le x \le \tau]$, $P[0 \le x \le 2\tau]$, ..., $P[0 \le x \le 5\tau]$ ($f(x) = 0 \quad x < 0$).

$$\int_0^\infty Ae^{-x/\tau}dx = \lim_{t \to \infty} \int_0^t Ae^{-x/t}\,dx =$$

$$= \lim_{t \to \infty} A\left(-\tau e^{-x/\tau}\Big|_0^t\right)$$

$$= \tau A = 1 \quad \therefore \quad \boxed{A = 1/\tau}$$

$$P(0 \le x \le \tau) = \int_0^\tau \frac{1}{\tau} e^{-x/\tau}dx = -e^{-x/\tau}\Big|_0^\tau = 1 - e^{-1} = 0.632.$$

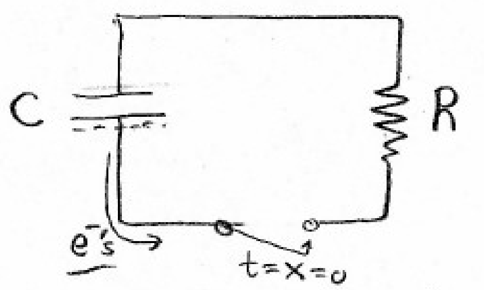
$$P(0 \le x \le 2\tau) = \int_0^{2\tau} \frac{1}{\tau} e^{-x/\tau}dx = 1 - e^{-2} = 0.865$$

$$P(0 \le x \le 3\tau) = 1 - e^{-3} = 0.950$$

$$P(0 \le x \le 4\tau) = 1 - e^{-4} = 0.982$$

$$P(0 \le x \le 5\tau) = 1 - e^{-5} = 0.993$$

This could model the chance a particular charge had flowed from a discharging capacitor ($\tau = RC$ in this case)



Typical values say

$R = 1k\Omega$

$C = 1\mu F$

$\tau = RC = 1ms$

$\boxed{\text{Def}^{n}/ \text{ The average value of } X \text{ is } \mu \equiv \int_{-\infty}^{\infty} x f(x)\,dx}$ also called mean value

This is just a continous weighted average, think about it. //

$\boxed{E2}$ What's average of last example

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx \quad : \quad f(x) = \begin{cases} 0 & x<0 \\ \frac{1}{\tau}e^{-x/\tau} & x \geq 0 \end{cases}$$

$$= \lim_{t\to\infty} \int_0^t \frac{x}{\tau} e^{-x/\tau}\,dx$$

Notice $\int \underset{u}{x} \underset{dV}{e^{ax}\,dx} = \frac{xe^{ax}}{a} - \int \frac{e^{ax}}{a}\,dx = \frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax}$ thus $a = -1/\tau$

$$\mu = \lim_{t\to\infty} \frac{1}{\tau}\left[-\tau x e^{-x/\tau} - \tau^2 e^{-x/\tau} \Big|_0^t \right]$$

$$= \lim_{t\to\infty} \frac{1}{\tau}\left[\underbrace{-\tau t e^{-t/\tau}} - \tau^2 \underset{\to 0}{e^{-t/\tau}} \right] + \lim_{t\to\infty} \left[\tau^2 e^0 \right]$$

$$\lim_{t\to\infty} \left(\frac{t}{e^{t/\tau}} \right) \overset{*}{=} \lim_{t\to\infty} \left(\frac{1}{\frac{1}{\tau}e^{t/\tau}} \right) = 0 \quad \text{(Need L-Hopital's Rule)}$$

$\therefore \boxed{\mu = \tau^2}$

$\boxed{\text{Def}^{n}/ \text{ The median value of } X \text{ is the \# } m \text{ such that } \int_m^{\infty} f(x)\,dx = \frac{1}{2}}$

$\boxed{E3}$ What's the median value of our example?

$$\frac{1}{2} = \int_m^{\infty} \frac{1}{\tau} e^{-x/\tau}\,dx = \lim_{t\to\infty} \int_m^t \frac{e^{-x/\tau}}{\tau}\,dx = \lim_{t\to\infty} \left(-\underset{\to 0}{e^{-t/\tau}} + e^{-m/\tau} \right) = e^{-m/\tau}$$

$$\frac{1}{2} = e^{-m/\tau} \Rightarrow \ln(1/2) = -m/\tau \quad \therefore \boxed{m = \tau \ln(2)} \quad (0.693\,\tau = m)$$

We skip Normal Distributions

(Really need multivariate calc. to do it right)