MA241-006: Calculus II

Instructor: Mr. James Cook
Test: #2 Form A
Date: Monday, Febuary 27, 2006

Directions: You **must** show **ALL** your work to receive credit.

1. (25 pts) Find the area of the region bounded by $y = x$ and $y = x^3$. For full credit your solution should include a graph and diagrams of the appropriate infinitesimal rectangle(s).

2. (25pts) Let us give the region bounded by $y = x^2$ and $y = \sqrt{x}$ the name *Dwight* . Find the volume of the solid obtained from rotating *Dwight* around the following:
   (i.) $x = 0$
   (ii.) $x = a, \quad a < 0$
   (iii.) $x = a, \quad a \geq 1$
   (iv.) $y = 0$
   (v.) $y = b, \quad b < 0$
   (vi.) $y = b, \quad b \geq 1$
   Do the above one at a time. On the test I'll pick one of the cases above. Again for full credit your solution should include a graph of the region as well as some picture of the typical approximating washer with r_{in} and r_{out} as they relate to the region in question (could be y_b or y_T or x_L or x_R as we have seen in examples). In other words, present your solution roughly as I have in class, try to indicate where the final integral came from. The set-up is worth most of the points on a problem like this.

3. (25 pts) Consider the curve in the xy-plane described parametrically by the equations:

$$\begin{aligned} x &= e^{\theta}(\sin\theta - \cos\theta) \\ y &= e^{\theta}(\sin\theta + \cos\theta) \\ 0 &\leq \theta \leq \ln 2 \end{aligned} \tag{1}$$

   Find the arclength of this curve. (If you forgot the formula for arclength you can buy it for 3pts during the test)

4. (25 pts) A well of depth h has a 1kg bucket which can carry 10kg of water when it is full. As the bucket is lifted from the base of the well to the surface it loses 4kg of water. Assume that the water leaks at a uniform rate from the bucket. Find the magnitude of work required to lift the bucket from the bottom of the well to the surface. You may leave your answer in terms of h and g, don't worry about the kg's.

5. (25 pts) Let $f(x) = kx^2(1-x)$ for $0 \leq x \leq 1$ and $f(x) = 0$ when $-\infty < x < 0$ or $1 < x < \infty$. Find what value we must assign to k if f(x) is to become a probabilty density function.