HOMEWORK QUIZ II : JULY 6 2007

§5.7#24 Integrate $\dfrac{x^2-x+6}{x^3+3x}$

$$\frac{x^2-x+6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$x^2-x+6 = A(x^2+3) + (Bx+C)x = (A+B)x^2 + Cx + 3A$$

Equating Coefficients of x^2, x and x^0 reveals,

$$\left.\begin{aligned} 1 &= A+B \\ -1 &= C \\ 6 &= 3A \end{aligned}\right\} \quad \begin{aligned} A &= 2 \\ C &= -1 \end{aligned} \quad \Rightarrow \quad B = 1-A = 1-2 = \underline{-1 = B}$$

thus,

$$\int \frac{x^2-x+6}{x^3+3x}\,dx = \int\left(\frac{2}{x} + \frac{-x}{x^2+3} + \frac{-1}{x^2+3}\right)dx$$

$$= 2\ln|x| - \underbrace{\int \frac{x\,dx}{x^2+3}}_{w=x^2+3} - \underbrace{\int \frac{dx}{x^2+3}}_{x=\sqrt{3}\tan\theta}$$

$$= 2\ln|x| - \frac{1}{2}\int \frac{dw}{w} - \int \frac{\sqrt{3}\sec^2\theta\,d\theta}{3\sec^2\theta}$$

$$= 2\ln|x| - \frac{1}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\theta + C$$

$$= \boxed{2\ln|x| - \frac{1}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C}$$

Supplemental #2

$$\int \sqrt{25-t^2}\,dt = \int \sqrt{25\cos^2 t}\cdot 5\cos t\,dt$$

$$\boxed{\begin{aligned} t &= 5\sin t \\ dt &= 5\cos t\,dt \\ 25-t^2 &= 25\cos^2 t. \end{aligned}}$$

$$= \int 25\cos^2 t\,dt$$

$$= \frac{25}{2}\int (1+\cos 2t)\,dt$$

$$= \frac{25}{2}\left(t + \frac{1}{2}\sin(2t)\right) + C$$

$$= \boxed{\frac{25}{2}\left[\sin^{-1}(t/5) + \frac{1}{2}\sin\left(2\sin^{-1}(t/5)\right)\right] + C}$$