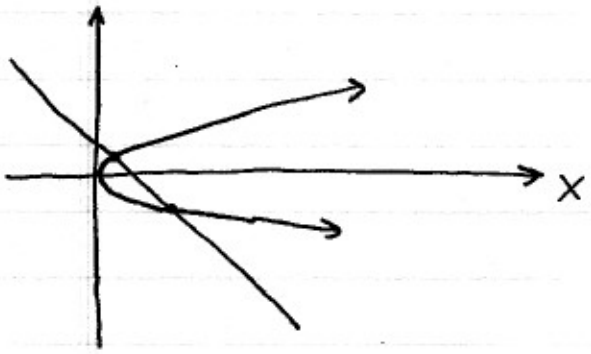
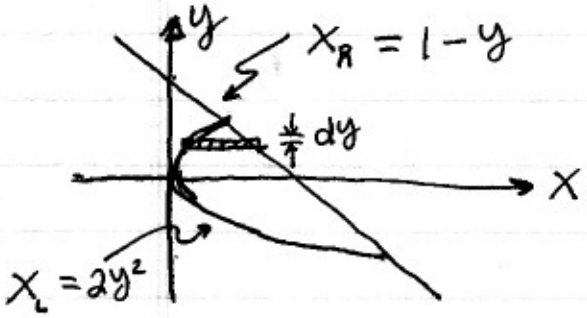
Homework Quiz III July 12, 2007

§6.1#11 Find area bounded by $x = 2y^2$ and $x + y = 1$



$$x_1 = x$$
$$2y^2 = 1 - y$$
$$2y^2 + y - 1 = 0$$
$$y = \frac{-1 \pm \sqrt{1+8}}{4} = -1, \frac{1}{2}.$$

lets draw it bigger,



$$dA = (x_R - x_L)\,dy$$
$$= (1 - y - 2y^2)\,dy$$

$$A = \int_{-1}^{1/2} (1 - y - 2y^2)\,dy = \left(y - \frac{y^2}{2} - \frac{2}{3}y^3\right)\Big|_{-1}^{1/2}$$

$$= \left(\frac{1}{2} - \frac{1}{8} - \frac{2}{24}\right) - \left(-1 - \frac{1}{2} + \frac{2}{3}\right)$$

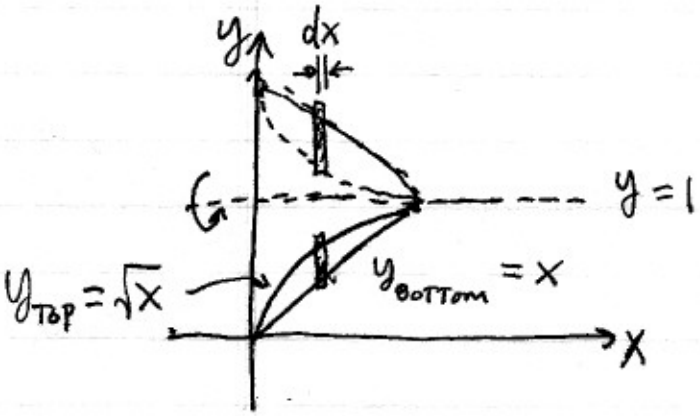
$$= \left(\frac{12 - 3 - 2}{24}\right) - \left(\frac{-24 - 12 + 16}{24}\right)$$

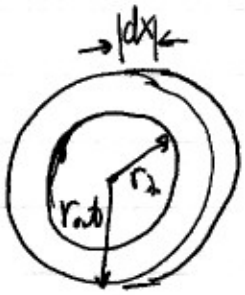
$$= \left(\frac{12 - 3 - 2 + 24 + 12 - 16}{24}\right)$$

$$= \left(\frac{48 - 21}{24}\right) = \frac{27}{24} = \boxed{\frac{9}{8}}$$

- there are of course better ways to do the arithmatic here.

§6.2#9 Find volume of solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about $y = 1$





$r_{in} = 1 - y_{TOP}$

$r_{out} = 1 - y_{BOTTOM}$

$$dV = \pi\left((1 - x)^2 - (1 - \sqrt{x})^2\right)dx$$

$$V = \int_0^1 \pi\left(1 - 2x + x^2 - 1 + 2\sqrt{x} - x\right)dx$$

$$= \int_0^1 \pi\left(x^2 + 2\sqrt{x} - 3x\right)dx$$

$$= \pi\left(\frac{1}{3} + 2\frac{1^{3/2}}{3/2} - \frac{3}{2}\right)$$

$$= \pi\left(\frac{1}{3} + \frac{4}{3} - \frac{3}{2}\right)$$

$$= \pi\left(\frac{10 - 9}{6}\right)$$

$$= \boxed{\pi/6}.$$

Remark: could also have tried cylindrical shells here