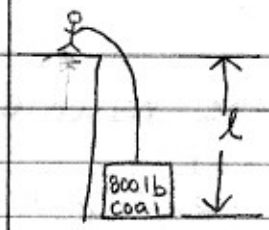
11.)

(§6.5)



Find the magnitude of work done.

$dW = (F_{gravity})\,d\ell$

$= (800 + 2\ell)$

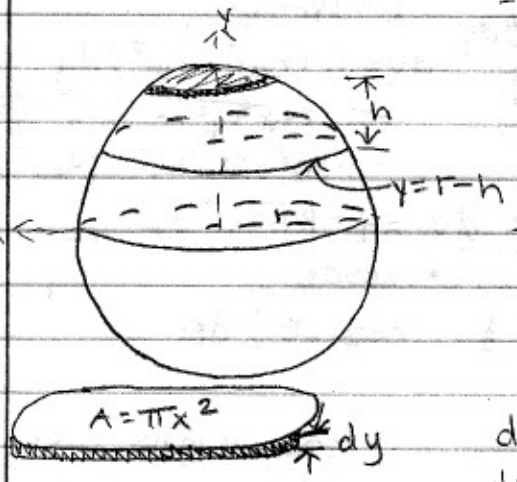
$W = \int_0^{500} (800 + 2\ell)\,d\ell$

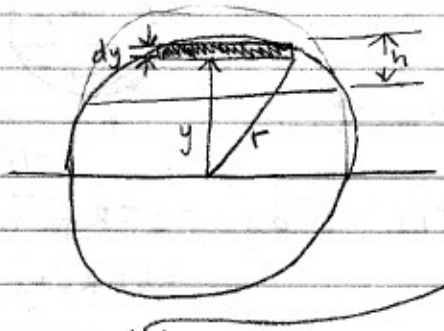
$= 800(500) + (500)^2$

$= \boxed{650{,}000 \text{ ft/lbs}}$

#27)

(§6.2)





$x^2 + y^2 = r^2$

$x^2 = r^2 - y^2$

$dv = \pi x^2\,dy$

$dv = \pi(r^2 - y^2)\,dy$

$V = \int_{r-h}^{r} \pi(r^2 - y^2)\,dy = \pi\left(r^2 dy = \frac{y^3}{3}\right)\Big|_{r-h}^{r}$

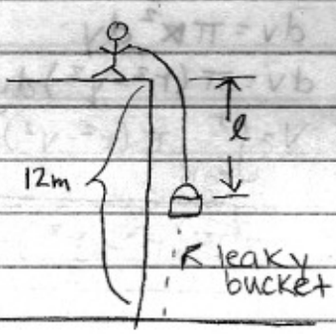
$= \boxed{\pi\left(r^2 - \frac{r^3}{3}\right) - \pi\left(r^2(r-h) - \frac{(r-h)^3}{3}\right)}$

$= W$

$\boxed{\quad = W}$

- for §6.2 # 39a see E6 on page 145 of my notes.

13 section 6.5



rope weighs 0.8kg/m
lifted at constant speed
3ℓ when ℓ=12
0 when ℓ=0

$M_w = (\text{slope})\ell + b$

$M_w(12) = 12s + b = 36$

$M_w(0) = b = 0$

$M_w = 3\ell$

$s = \frac{3b}{12}$

$F_{gravity} = M_{rope}g + M_{water+bucket}g$

$= 0.8\ell g + 3\ell g$

$= 3.8\ell g$

$dW = (3.8\ell g)d\ell \Rightarrow W = \int_{12}^{0} (3.8\ell g)d\ell$

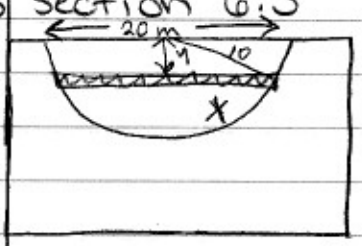
$W = \frac{3.8}{2}g(\ell^2)\Big|_{12}^{0}$

$0 - \frac{3.8}{2}\cdot\frac{24}{1}g$

$= -456g$

$|W| = 456g.$

#23 section 6.5



Find hydrostatic force on half-circle gate, assuming water behind the gate up to the top.

$x^2+y^2=100$

$x=\sqrt{100-y^2}$

$dA = 2x\,dy$

$P = \rho g d = \rho g y$

$P = \frac{dF}{dA} \Rightarrow dF = P\,dA = \rho g y \cdot 2x\,dy = 2\rho g y\sqrt{100-x^2}\,dy$

$u = 100-y^2$

$du = -2y\,dy$

$F = \int_0^{10} 2\rho g y\sqrt{100-y^2}\,dy$

$= \int_{y=0}^{y=10} -\rho g\sqrt{u}\,du$

$= -\frac{2\rho g}{3} u^{3/2}\Big|_{y=0}^{y=10}$

$= -\frac{2\rho g}{3}(100-y^2)^{3/2}\Big|_0^{10}$

$= \frac{2\rho g}{3}(100)^{3/2}$

$= \boxed{666.\overline{6}\,\rho g}$ newtons