QUIZ 5: FIRST ORDER ODEs (7.3#7.3+), 7.3#15, 7.3+#9

Remark: we threw out 7.3+ #10 due to the integral at the end.

7.3#15 Find an eqⁿ of the curve satisfying $\frac{dy}{dx} = 4x^3y$ and whose y-intercept is 7. First separate,

$$\frac{dy}{dx} = 4x^3y \Rightarrow \frac{dy}{y} = 4x^3\,dx$$

$$\Rightarrow \int \frac{dy}{y} = \int 4x^3\,dx$$

$$\Rightarrow \ln|y| = x^4 + C$$

Now we can solve for $|y| = \exp(x^4 + C) = \exp(x^4)\exp(C)$ thus $y = \pm e^C e^{x^4}$. Now use the initial condition,

$$y(x=0) = 7 = \pm e^C e^{0^4} = \pm e^C \quad \therefore \quad \pm e^C = 7$$

$$\therefore \boxed{y = 7e^{x^4}}$$

§7.3+#9 (from the additional topics pdfs)

$$xy' + y = \sqrt{x} \quad (\text{assume } x > 0)$$

$$y' + \left(\frac{1}{x}\right)y = \frac{1}{x}\sqrt{x}$$

$$I = \exp\left(\int \frac{1}{x}dx\right) = \exp(\ln|x|) = |x| = x \quad (x>0)$$

$$xy' + x\frac{1}{x}y = x\frac{1}{x}\sqrt{x}$$

$$xy' + y = \sqrt{x}$$

(full circle, this problem was designed so you could start with the product rule, its silly.)

$$\int \frac{d}{dx}(xy)\,dx = \int \sqrt{x}\,dx$$

$$xy = \frac{2}{3}x^{3/2} + C$$

$$\therefore \boxed{y = \frac{2}{3}\sqrt{x} + \frac{C}{x}}$$