# Quiz 6: Nonhomogeneous $2^{nd}$ order ODEs (7.8 # 3, 9)

§7.8 #3 find general sol$^n$ to $y'' - 2y' = \sin(4x)$

① $\lambda^2 - 2\lambda = \lambda(\lambda - 2) = 0 \therefore \lambda_1 = 0, \lambda_2 = 2 \Rightarrow y_h = c_1 + c_2 e^{2x}$

② $y_p = A\sin(4x) + B\cos(4x)$ {no overlap here ☺}

③ $y_p' = 4A\cos(4x) - 4B\sin(4x)$

$y_p'' = -16A\sin(4x) - 16B\cos(4x)$

④ $y_p'' - 2y_p' = -16A\sin(4x) - 16B\cos(4x) - 2(4A\cos 4x - 4B\sin 4x)$

$\Rightarrow \sin(4x) = (-16B - 8A)\cos(4x) + (-16A + 8B)\sin(4x)$

Equate Coeff.

$\sin 4x: \quad 1 = -16A + 8B$

$\cos 4x: \quad 0 = -16B - 8A \Rightarrow A = -2B \qquad 1 = 40B$

$\left.\right\} A = -1/20, \; B = 1/40$

⑤ $\boxed{y = y_h + y_p = c_1 + c_2 e^{2x} - \frac{1}{20}\sin(4x) + \frac{1}{40}\cos(4x)}$

§7.8 #9 Find sol$^n$ to $y'' - y' = xe^x$ with initial conditions $y(0) = 2$ and $y'(0) = 1$.

① $\lambda^2 - \lambda = \lambda(\lambda - 1) = 0 \therefore \lambda_1 = 0 \;\&\; \lambda_2 = 1$

$\therefore y_h = c_1 + c_2 e^x$

② $y_{p(naive)} = Axe^x + Be^x$ (overlaps $e^x$ so ↯)

$y_p = Ax^2e^x + Bxe^x$ this should work.

Remark: finding the correct choice for $y_p$ is a little tricky. Consult the examples, hwk and also pg. 190f (you can ignore 1. and 12. I don't require knowledge of the hyperbolic trig. facts for ma 241-003 summer II)

Continued ↲

$\boxed{\text{§7.8 \#9, Continued}}$

(3) $Y_p = e^x(Ax^2 + Bx)$

$Y_p' = e^x(Ax^2 + Bx) + e^x(2Ax + B)$

$= e^x(Ax^2 + (2A+B)x + B)$

$Y_p'' = e^x(Ax^2 + (2A+B)x + B) + e^x(2Ax + (2A+B))$

$= e^x(Ax^2 + (4A+B)x + 2B + 2A$

(4) $Y_p'' - Y_p' = xe^x$

$e^x(Ax^2 + (4A+B)x + 2B+2A) - e^x(Ax^2 + (2A+B)x + B) = xe^x$

$e^x\left(x^2[A-A] + x[4A+\cancel{B} - 2A - \cancel{B}] + 2B+2A - B\right) = xe^x$

Equate Coeff.

$xe^x$ : $2A = 1$ $\quad \therefore \quad A = 1/2$.

$e^x$ : $B + 2A = 0$ $\quad B = -2A = \frac{-2}{2} = -1.$

(5) $Y = Y_h + Y_p = c_1 + c_2e^x + \frac{1}{2}x^2e^x - xe^x$

this is the general sol$^{n}$. If we had no initial conditions then this would be all we could do

(6) However $y(0) = 2$ and $y'(0) = 1$ here so,

$2 = c_1 + c_2 \leftarrow$ (used $y(0) = 2$)

And note $y' = c_2e^x + \frac{1}{2}(2xe^x + x^2e^x) - e^x - xe^x$

thus $y'(0) = 1$ says that

$1 = c_2 - 1 \Rightarrow \boxed{c_2 = 2} \Rightarrow c_1 = 2 - c_2$

$\Rightarrow \boxed{c_1 = 0}$

Collecting our results we find,

$$\boxed{Y = 2e^x + \frac{1}{2}x^2e^x - xe^x}$$