MA241-007: Calculus II

Instructor: Mr. James Cook
Test: #1 Form A
Date: Monday, Febuary 7, 2005

Directions: You **must** show **ALL** your work to receive credit. Please place a circle or box around your final answer for each part.

1. (20 pts) Partial Fractions

(a) Set-up (that is don't find A,B,... etc.) the partial fractional decomposition for:

$$\frac{x^3 - 3x - 1}{x(x^2+4)(x+3)^2(x^2+1)^2}$$

$$= \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{D}{x+3} + \frac{E}{(x+3)^2} + \frac{Fx+G}{x^2+1} + \frac{Hx+I}{(x^2+1)^2}$$

(b) Use partial fractions to calculate the integral below:

$$\int \frac{x+4}{x^2+5x+6}\,dx = \int \frac{x+4}{(x+3)(x+2)} = I$$

$$\frac{x+4}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$$

$$x+4 = A(x+2) + B(x+3)$$

$\underline{x=-2)}$ $2 = B$

$\underline{x=-3)}$ $1 = -A$

$$I = \int\left(\frac{-1}{x+3} + \frac{2}{x+2}\right)dx$$

$$= \boxed{-\ln|x+3| + 2\ln|x+2| + c}$$

0

2. (44 pts) Calculate the following indefinite integrals. Clearly indicate any substitutions you make.

(a) $\int x^3(1-x^4)^{19}\,dx = \int x^3 u^{19} \frac{du}{-4x^3}$ ⟵ $u = 1 - x^4$, $du = -4x^3\,dx$, $dx = \frac{du}{-4x^3}$

$= \frac{-1}{4}\int u^{19}\,du$

$= \frac{-1}{4}\frac{u^{20}}{20} + C$

$= \boxed{\frac{-1}{80}(1-x^4)^{20} + C}$

(b) $\int \overbrace{x^5 \underbrace{\ln(x)}_{u}\,dx}^{dV} = \frac{x^6}{6}\ln(x) - \int \frac{x^6}{6}\frac{dx}{x}$ ⟵ $u = \ln(x)$, $du = dx/x$, $dV = x^5\,dx$, $V = \frac{1}{6}x^6$

$= \frac{1}{6}x^6\ln(x) - \frac{1}{6}\int x^5\,dx$

$= \frac{1}{6}x^6\ln(x) - \frac{1}{36}x^6 + C$

$= \boxed{\frac{1}{6}x^6\left(\ln(x) - \frac{1}{6}\right) + C}$

(c) $\int \sin^3(x)\,dx = \int \sin^2(x)\sin(x)\,dx$

$= \int (1-\cos^2(x))\sin(x)\,dx$

$= \int (1-u^2)(-du)$ $\leftarrow$ $\boxed{\begin{array}{l} u = \cos(x) \\ du = -\sin(x)\,dx \end{array}}$

$= \int (u^2-1)\,du$

$= \frac{u^3}{3} - u + C = \boxed{\frac{\cos^3(x)}{3} - \cos(x) + C}$

(d) $\int \frac{1}{(x^2-1)^{3/2}}\,dx = \int \frac{\sec(\theta)\tan\theta\,d\theta}{(\tan^2\theta)^{3/2}}$

$x = \sec(\theta)$

$x^2-1 = \sec^2(\theta) - 1 = \tan^2(\theta)$

$\left(\begin{array}{l} \sin^2(\theta) + \cos^2(\theta) = 1 \\ \tan^2(\theta) + 1 = \sec^2(\theta) \end{array}\right)$

$dx = \sec(\theta)\tan(\theta)\,d\theta$

$= \int \frac{\sec(\theta)\tan\theta}{\tan^3(\theta)}\,d\theta$

$= \int \sec\theta \frac{1}{\tan^2\theta}\,d\theta$

$= \int \frac{1}{\cos\theta}\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$

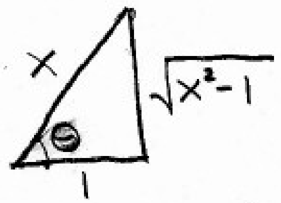
$= \int \frac{\cos\theta}{\sin^2\theta}\,d\theta$

$= \int \frac{du}{u^2}$ $\leftarrow$ $\boxed{\begin{array}{l} u = \sin\theta \\ du = \cos\theta\,d\theta \end{array}}$

$= \frac{-1}{u} + C$

$= \frac{-1}{\sin\theta} + C$

$x = \sec\theta = \frac{\text{hyp}}{\text{adj}}$



$= \boxed{\frac{-x}{\sqrt{x^2-1}} + C}$

$\frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}} = \frac{x}{\sqrt{x^2-1}}$

3. (20 pts) Evaluate the following definite integrals. The answers should involve only numbers, no functions for full credit.

(a) $\int_0^1 \sin(\pi x)\,dx = \int_0^{\pi} \sin(u)\,\frac{du}{\pi}$

$u = \pi x \quad u(0) = \pi \cdot 0 =$

$du = \pi dx \quad u(1) = \pi \cdot 1 = \pi$

$dx = \frac{du}{\pi}$

$= \frac{-1}{\pi} \cos(u)\Big|_0^{\pi}$

$= \frac{-1}{\pi}(\cos(\pi) - \cos(0))$

$= \frac{-1}{\pi}(-1-1)$

$= \boxed{\frac{2}{\pi}}$

(b) $\int_2^3 \frac{1}{(x^2-1)^{3/2}}\,dx = \frac{-x}{\sqrt{x^2-1}}\Big|_2^3$

$= \boxed{\frac{-3}{\sqrt{8}} + \frac{2}{\sqrt{3}}}$

4. (15 pts) Let $v(t) = t^3$ be the velocity of a particle at time t in meters per second.

(a) Calculate $\int_0^4 v(t)\,dt$. What does this integral tell you (one sentence)?

$$\int_0^4 V(t)dt = \int_0^4 t^3 dt = \frac{1}{4}t^4\Big|_0^4 = \frac{1}{4}4^4 = 4^3 = \boxed{64m}$$

This is the displacement during the first four seconds, bc, by FTC,

$$\int_0^4 \frac{dx}{dt}\,dt = x(4) - x(0)$$

(b) Find M_2 to approximate the displacement during the first 4 seconds.

$\Delta t = \frac{4}{2} = 2 \qquad t_0 = 0,\ t_1 = 2,\ t_2 = 4$

$\bar{t}_1 = 1$ & $\bar{t}_2 = 3$ thus

$$M_2 = 2(\bar{t}_1^3 + \bar{t}_2^3) = 2(1^3 + 3^3) = 2(28) = \boxed{56m}$$

(c) Find an upperbound K such that $|v''(t)| \leq K$ when $0 \leq t \leq 4$. Then use that $|E_{M_n}| \leq \frac{64K}{24n^2}$ to find an upperbound on the error in M_2. Is this a reasonable upperbound given what you know from your earlier calculations in this problem?

$V(t) = t^3$

$V'(t) = 3t^2$

$V''(t) = 6t$

$V'''(t) = 6 \Rightarrow V''$ increasing on $[0,4]$

thus $V''(0) \leq V''(t) \leq V''(4) = 6(4) = \boxed{24}$

$$|E_{M_2}| \leq \frac{64(\cancel{24})}{\cancel{24}(2)^2} = \frac{64}{4} = 16$$

Clearly $E_{M_2} = I - M_2 = 64 - 56 = 8 < 16$

So it's a reasonable upper-bound, it's actually pretty generous.