§9.5 #9 Find parametric & symmetric eq$^{\underline{n}}$'s for given line, through $(1,-1,1)$ and parallel to $x+2 = \frac{1}{2}y = z-3$.
My preference is to find a vector along the line so just pick two points, $x=0$ and $x=1$

$$\underline{x=0} \Rightarrow 2 = \tfrac{1}{2}y \Rightarrow \underline{y=4} \Rightarrow 2 = z-3 \Rightarrow \underline{z=5}$$

$$\underline{x=1} \Rightarrow 3 = \tfrac{1}{2}y \Rightarrow \underline{y=6} \Rightarrow 3 = z-3 \Rightarrow \underline{z=6}$$

Thus $A = (0,4,5)$ and $B = (1,6,6)$ are on the line hence $AB = \langle 1,2,1\rangle$ is direction vector for line. Thus,

$$r(t) = \langle 1,-1,1\rangle + t\langle 1,2,1\rangle$$
$$= \langle 1+t,\ -1+2t,\ 1+t\rangle$$

Therefore we find

$$\boxed{\begin{aligned} x &= 1+t \\ y &= -1+2t \\ z &= 1+t \end{aligned}} \qquad \begin{aligned} &\Rightarrow t = x-1 \\ &\Rightarrow t = \tfrac{1}{2}(y+1) \\ &\Rightarrow t = z-1 \end{aligned} \Bigg\} \Rightarrow \boxed{x-1 = \tfrac{1}{2}(y+1) = z-1}$$

parametric eq$^{\underline{n}}$'s for the line.

Symmetric eq$^{\underline{n}}$'s for the line.

__Remark:__ the answers here are not unique. Notice I just picked two values for x out of convenience. Different choices would have given different A & B. Honestly I'm surprised we got the book's answer here. In general there are infinitely many parametrizations of a given line (or curve or plane etc...)

§9.5#16 Find parametric eq$^{\underline{n}}$'s for line segment joining $(10,3,1)$ to $(5,6,-3)$. We construct,

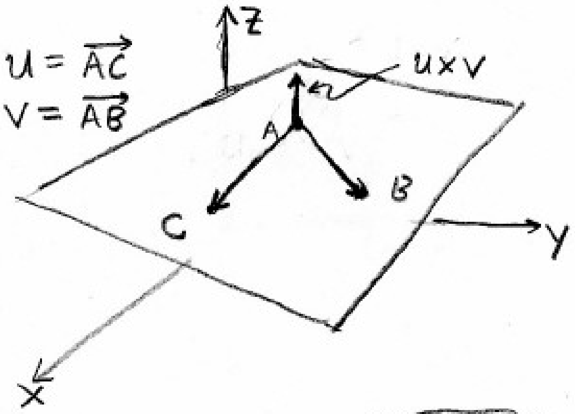
$$r(t) = (1-t)(10,3,1) + t(5,6,-3)$$
$$= \langle 10-10t+5t,\ 3-3t+6t,\ 1-t-3t\rangle$$

Thus $\boxed{x = 10-5t,\ y = 3+3t,\ z = 1-4t,\ 0 \le t \le 1}$

- notice we can check it works, try $t=0$ you'll get $(10,3,1) = r(0)$ whereas for $t=1$ you'll get $r(1) = (5,6,-3)$. __Check your work!__

$\boxed{\S 9.5 \# 25}$ Find plane through the points $(0,1,1) = A$ and $B = (1,0,1)$ and $C = (1,1,0)$. Pictorially

$u = \overrightarrow{AC}$
$v = \overrightarrow{AB}$



$$u = \overrightarrow{AC} = C - A = \langle 1, 0, -1\rangle$$
$$v = \overrightarrow{AB} = B - A = \langle 1, -1, 0\rangle$$
$$u = \hat{i} - \hat{k}, \quad v = \hat{i} - \hat{j}$$

$$\boxed{\begin{aligned} u \times v &= (\hat{i} - \hat{k}) \times (\hat{i} - \hat{j}) \\ &= -\hat{i} \times \hat{j} - \hat{k} \times \hat{i} + \hat{k} \times \hat{j} \\ &= -\hat{k} - \hat{j} - \hat{i} \\ &= \langle -1, -1, -1\rangle \end{aligned}}$$

Can also do the other way

Now I have a normal vector and a point (well three actually, let's use A.) thus the plane is

$$-1(x-0) - (y-1) - (z-1) = 0$$
$$\boxed{x + y + z = 2}$$

Let's check our work, $0+1+1 = 2$, $1+0+1 = 2$, $1+1+0 = 2$ all three points lie on the plane as needed.

$\boxed{\S 9.5 \# 30}$ Find plane through intersection of $x - z = 1$ & $y + 2z = 3$ that is also perpendicular to $x + y - 2z = 1$.

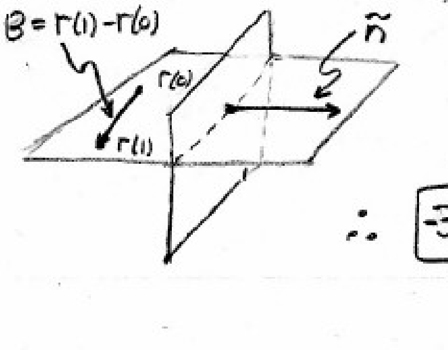
intersection $\Rightarrow$ both eq^{n}'s hold $\Rightarrow x - z - 1 = 0 = y + 2z - 3$

$\Rightarrow x = z + 1$ and $y = 3 - 2z$

$\Rightarrow r(z) = (z+1, 3-2z, z)$: use z as parameter

$\Rightarrow \left.\begin{aligned} r(0) &= (1,3,0) \\ r(1) &= (2,1,1) \end{aligned}\right\}$ points on our plane.

We want our plane perpendicular to $x + y - 2z = 1$ which has normal $\langle 1, 1, -2\rangle \equiv \tilde{n}$. We have two vectors on our plane. provided they're not parallel we're almost done.



$$\tilde{n} \times (r(1) - r(0)) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = \langle -3, -3, -3\rangle$$

normal to our plane

$$\therefore \boxed{-3(x-1) - 3(y-3) - 3z = 0}$$

used $r(0)$

§9.5 # 35 Find angle between the given planes,

$$x - 4y + 2z = 0 \quad \Rightarrow \quad n_1 = \langle 1, -4, 2 \rangle$$
$$2x - 8y + 4z = -1 \quad \Rightarrow \quad n_2 = \langle 2, -8, 4 \rangle$$

$$n_1 \cdot n_2 = |n_1||n_2| \cos\theta = \sqrt{1+16+4}\sqrt{4+64+16}\cos\theta = 2 + 32 + 8$$

$$\Rightarrow \cos\theta = \frac{42}{\sqrt{21}\sqrt{84}} = \frac{42}{\sqrt{21}\sqrt{4(21)}} = \frac{42}{21\sqrt{4}} = \frac{42}{42} = 1.$$

$\Rightarrow \boxed{\theta = 0}$ they are parallel.

We could have simply noted $\underline{n_2 = 2n_1}$. Anyway this sol^{n} still works for all the other cases.

§9.6 # 3 Let $f(x,y) = x^2 e^{3xy}$. We find,

(a.) $f(2,0) = 2^2 e^0 = \boxed{4}$

(b.) $\text{dom}(f) = \{(x,y) \mid x, y \in \mathbb{R}\} = \boxed{\mathbb{R}^2}$ since polynomials and the exponential are continuous everywhere.

(c.) $\text{range}(f) = \{z = f(x,y) \mid (x,y) \in \text{dom}(f)\}$

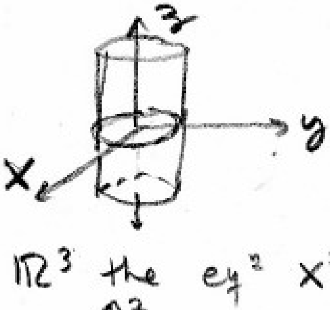
notice $x^2 \geq 0$ and $e^u > 0$ $\forall x$ and u. Thus

$\boxed{\text{range}(f) = [0, \infty)}$

§9.6 # 23 The interpretation of eq^{n}'s varies depending where they live.

(a.) In $\mathbb{R}^2$ the eq^{n} $x^2 + y^2 = 1$ is a circle centered at the origin with radius one.

(b.) In $\mathbb{R}^3$ the eq^{n} $x^2 + y^2 = 1$ is a cylinder along z-axis.



notice each plane $y = k$ slices the cylinder to give a circle.

(c.) In $\mathbb{R}^3$ the eq^{n} $x^2 + z^2 = 1$ gives a circle at each y, its a cylinder of radius one centered on y-axis.

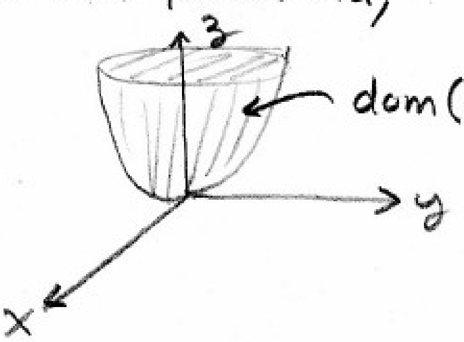
$\boxed{\S 11.1 \# 7}$ Let $f(x,y,z) = \exp(\sqrt{z - x^2 - y^2})$

(a.) $f(2,-1,6) = \exp(\sqrt{6-4-1}) = \boxed{\exp(1) \cong 2.71}$

(b.) the domain (f) is determined by the square root function, we need the inputs $z - x^2 - y^2 \geq 0$.

$$z \geq x^2 + y^2 \quad \text{a.k.a} \quad x^2 + y^2 \leq z$$

this shape begins at the origin $(0,0,0)$, since $x^2+y^2 \geq 0$ we cannot have $z < 0$ then we can take a slice at $z = k$ we have locus of all points with $x^2+y^2 \leq k$, this is a disk of radius $\sqrt{k}$ at $z = k$. On the (xz) or (yz) planes we have $z \geq x^2$ ($y=0$ on xz-plane) and $z \geq y^2$ ($x = 0$ on yz-plane) this is a solid paraboloid,



(c.) Clearly if $x=0, y=0$ then $f(0,0,z) = e^{\sqrt{z}}$ and z may get arbitrarily large in dom(f) so $\boxed{\text{range}(f) = [1, \infty)}$

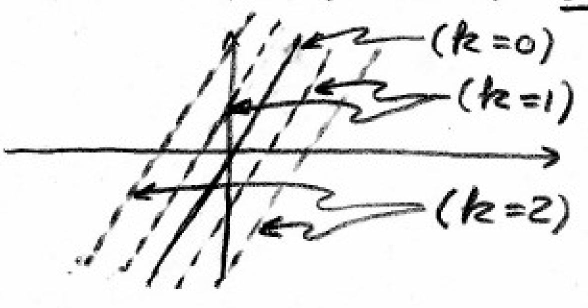
$\boxed{\S 11.1 \# 15}$ Let $f(x,y) = (y-2x)^2$ find some level curves, make contour map,

$f(x,y) = 0 = (y-2x)^2 \Rightarrow y - 2x = \pm 0 \quad \therefore y = 2x$

$f(x,y) = 1 = (y-2x)^2 \Rightarrow y - 2x = \pm 1 \quad \therefore y = 2x+1 \text{ OR } y = 2x-1$

$f(x,y) = k = (y-2x)^2 \Rightarrow y - 2x = \pm k \quad \therefore y = 2x+k \text{ OR } y = 2x-k$

the level surface $f(x,y) = k$ is <u>disconnected</u> into two lines. (except for $k = 0$)



<u>remark</u>: the graph $z = f(x,y)$ would be a triangular valley.

$\boxed{\S 11.1 \# 31}$ $z = \sin(xy)$ is the graph of $f(x,y) = \sin(xy)$. Match this graph with the figures on pg. 749.

(a.) notice along $x = 0$ we have $z = \sin(0) = 0$
likewise along $y = 0$ we have $z = \sin(0) = 0$
hmm... if $x = 1$ we have $z = \sin(y)$
$x = 2$ we have $z = \sin(2y)$
$x = 3$ we have $z = \sin(3y)$
$x = -1$ we get $z = \sin(-y) = -\sin(y)$
for large values it oscilates quickly.

Let's see on $[-\pi, \pi] \times [-\pi, \pi]$ we have
$\sin(xy) > 0$ for $0 < x, y < \pi$
$\sin(xy) > 0$ for $-\pi < x, y < 0$
$\sin(xy) < 0$ for $-\pi < x < 0$ and $0 < y < \pi$
$\sin(xy) < 0$ for $0 < x < \pi$ and $-\pi < y < 0$.
Also $-1 \leq z \leq 1$ since $|\sin(xy)| \leq 1$.

Upon consideration of this data we choose C and II.

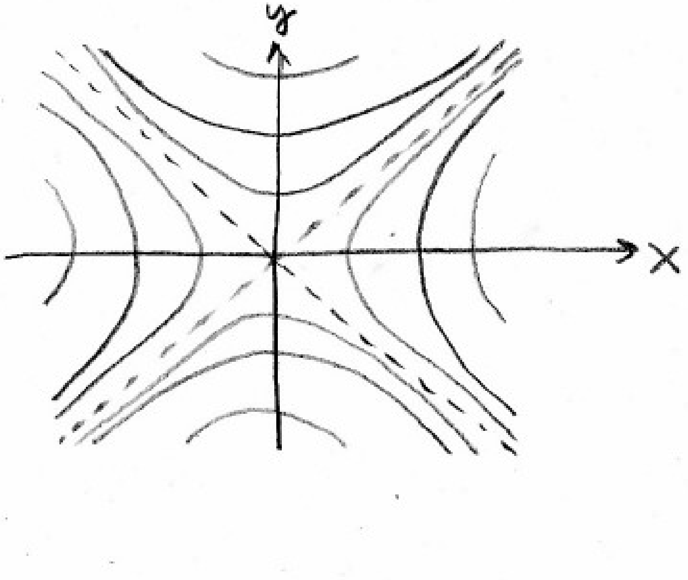
Remark: its probably best to use a mixture of Maple and analytic observations to determine important features of a graph $z = f(x,y)$.

$\boxed{\S 11.1 \# 40}$ What are level surfaces of $f(x,y,z) = x^2 - y^2$

$$f(x,y,z) = k = x^2 - y^2$$

There are three main cases

(i) $k = 0$ then $x^2 = y^2 \Rightarrow y = \pm x$
(ii) $k > 0$ then $x^2 - y^2 = k$ (hyperbola opens sideways)
(iii) $k < 0$ then $y^2 - x^2 = -k$ (hyperbola opens up/down)



now where is z in all of this ?
We note this picture happens at each z.
The level-surfaces are gotten from extending the graph into/out-of page.