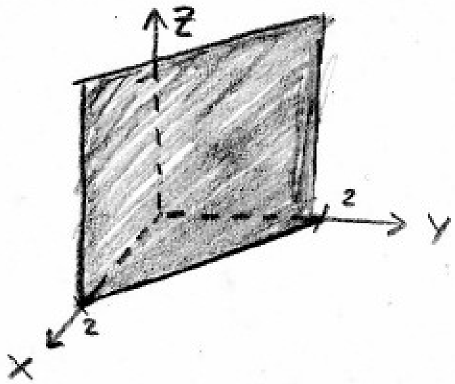
REQUIRED HOMEWORK SOLUTIONS FOR TEST I

I solve problems from early sections with the knowledge of later sections. This means my sol$^{\underline{n}}$'s are not necessarily what your text intentioned, but they should be close to how you would solve them since you know more, ...

$\boxed{\S 9.1\#5}$ Describe and sketch $x+y=2$. We identify that this is a plane with normal

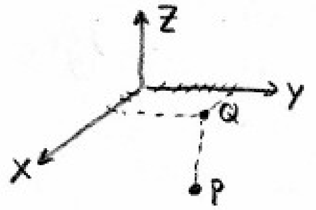


$$n = \langle 1, 1, 0 \rangle$$

we can write $y = 2 - x$ which suggests the intersection line with the xy-plane I've drawn.

$\boxed{\S 9.1\#8}$ find distance from $(3,7,-5) \equiv P$ to the following,

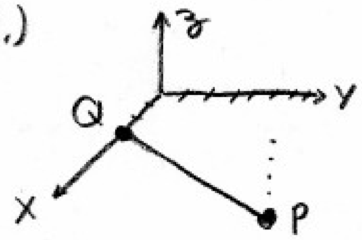
(a.) The xy-plane, $|\underbrace{(3,7,-5)}_{P} - \underbrace{(3,7,0)}_{Q}| = |(0,0,-5)| = \boxed{5}$



(b.) Same principle as in (a.), distance to yz-plane is the absolute value of x-coordinate. $\boxed{3}$

(c.) likewise $\boxed{7}$ is distance to $y=0$ plane.

(d.)



the closest point Q should be a perpendicular line to the x-axis, it follows geometrically $Q = (3,0,0)$

$$|P-Q| = |(3,7,-5) - (3,0,0)|$$
$$= |(0,7,-5)|$$
$$= \sqrt{49+25} = \boxed{\sqrt{74}} \cong 8.6$$

(e.) $|P-Q| = |(3,7,-5) - (0,7,0)| = |(3,0,-5)| = \sqrt{9+25} = \boxed{\sqrt{34}} \cong 5.83$

(f.) $|P-Q| = |(3,7,-5) - (0,0,-5)| = |(3,7,0)| = \sqrt{9+49} = \boxed{\sqrt{58}} \cong 7.62$

$\boxed{\S 9.1 \# 10}$ The sphere centered at $(2,-6,4) = C$ and radius 5 is the set of all points (x,y,z) which are 5 units away from C,

$$|(x,y,z)-(2,-6,4)| = 5 \Rightarrow \boxed{(x-2)^2+(y+6)^2+(z-4)^2 = 25}$$ Call these points S

The xy-plane has $z=0$, the intersection with S has,

$$(x-2)^2+(y+6)^2 = 25-16 = 9$$

it's a circle of radius 3 centered at $(2,-6,0)$ on $z=0$

The yz-plane is $x=0$, the intersection with S satisfies,

$$(y+6)^2+(z-4)^2 = 25-4 = (\sqrt{21})^2$$

it's a circle of radius $\sqrt{21}$ at $(0,-6,4)$ on $x=0$

The zx-plane has $y=0$ which gives the intersection points

$$(x-2)^2+(z-4)^2 = 25-36 = -11 < 0$$

BUT $(x-2)^2+(z-4)^2 \geq 0$ $\therefore$ the intersection is empty, that is our sphere S does not intersect the zx-plane.

$\boxed{\S 9.1 \# 16}$ Find the eq$^{\underline{n}}$ of the sphere with a diameter whose endpoints are $(2,1,4)$ and $(4,3,10)$. Geometrically we observe the midpoint of any diameter is the center of the sphere. The midpoint is found via

$$C = \frac{1}{2}\left[(2,1,4)+(4,3,10)\right] = \frac{1}{2}(6,4,14) = (3,2,7).$$

The radius will be half the length of the diameter

$$\text{radius} = \frac{1}{2}|(4,3,10)-(2,1,4)| = \frac{1}{2}|(2,2,6)| = \frac{1}{2}\sqrt{4+4+36} = \sqrt{11}$$

$$\therefore \boxed{(x-3)^2+(y-2)^2+(z-7)^2 = 11}$$

§9.1#35 Find all points (x, y, z) equidistant from the points $(-1, 5, 3)$ and $(6, 2, -2)$. That is

$$|(x,y,z)-(-1,5,3)| = |(x,y,z)-(6,2,-2)|$$

$$\Rightarrow |(x+1, y-5, z-3)|^2 = |(x-6, y-2, z+2)|^2$$

$$\Rightarrow (x+1)^2 + (y-5)^2 + (z-3)^2 = (x-6)^2 + (y-2)^2 + (z+2)^2$$

$$\Rightarrow (x^2+2x+1) + (y^2-10y+25) + (z^2-6z+9) + $$

$$- (x^2-12x+36) - (y^2-4y+4) - (z^2+4z+4) = 0$$

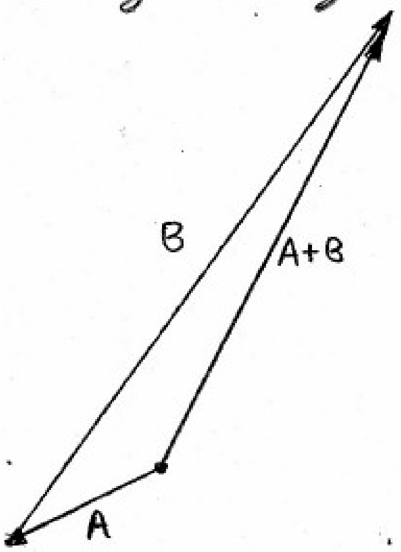
$$\Rightarrow 14x - 35 - 6y + 21 - 10z + 5 = 0$$

$$\Rightarrow \boxed{14(x - 35/14) - 6(y + 21/6) - 10(z + 5/10) = 0}$$

this is a plane through $(35/14, -21/6, -5/10)$ with normal $\langle 14, -6, -10 \rangle$.

§9.2#12 Let $A = \langle -2, -1 \rangle$ and $B = \langle 5, 7 \rangle$

we calculate algebraically $A + B = \langle -2, -1 \rangle + \langle 5, 7 \rangle = \langle 3, 6 \rangle$.



tip-to-tail

• I'd provide graph paper for this sort of question on the test

§9.2#16 Let $a = 4\hat{i} + \hat{j} = \langle 4, 1\rangle$ and $b = \hat{i} - 2\hat{j} = \langle 1, -2\rangle$.

$$a+b = \langle 4,1\rangle + \langle 1,-2\rangle = \langle 4+1, 1-2\rangle = \boxed{\langle 5, -1\rangle = a+b}$$

$$2a+3b = 2\langle 4,1\rangle + 3\langle 1,-2\rangle = \langle 8,2\rangle + \langle 3,-6\rangle = \boxed{\langle 11,-4\rangle = 2a+3b}$$

$$|a| = \sqrt{4^2+1^2} = \sqrt{16+1} = \boxed{\sqrt{17} = |a|}$$

$$|a-b| = |\langle 4,1\rangle - \langle 1,-2\rangle| = |\langle 3,3\rangle| = \sqrt{9+9} = \boxed{\sqrt{18} = |a-b|}$$

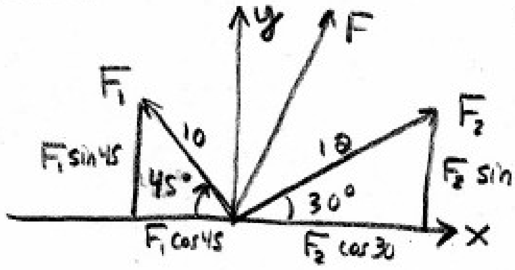
§9.2#20 Find W in the same direction as $\langle -2, 4, 2\rangle$ but with $|W| = 6$. This means $W = k\langle -2,4,2\rangle$ for some $k \neq 0$ constant.

$$|W| = 6 = |k\langle -2,4,2\rangle| = |k|\,|\langle -2,4,2\rangle| = |k|\sqrt{4+16+4}$$

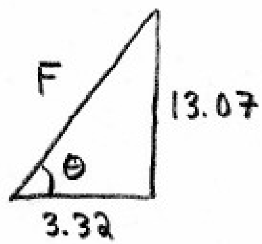
$$\Rightarrow |k| = \frac{6}{\sqrt{24}} = \frac{6}{2\sqrt{6}} = \frac{3}{\sqrt{6}} \quad \therefore k = \pm 3/\sqrt{6}$$

we choose $k = 3/\sqrt{6}$ because we want W to have same direction, thus $\boxed{W = \frac{3}{\sqrt{6}}\langle -2, 4, 2\rangle}$ or $\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6}\rangle$ if you prefer.

§9.2#23 Find $F = F_1 + F_2$ and the angle of F relative to the x-axis and $|F|$



	x	y
F_1	-7.07	7.07
F_2	10.39	6
F_1+F_2	3.32	13.07



$$\tan\theta = \frac{13.07}{3.32} \quad \therefore \quad \theta = \tan^{-1}\left(\frac{13.07}{3.32}\right) = \boxed{75.7°}$$

$$|F| = \sqrt{(3.32)^2 + (13.07)^2} \cong \boxed{13.49 = |F|}$$

- Moral of Story: break the vectors up into their <u>vector</u> <u>components</u>.

§9.2#24 Basically same ideas as #23.

9.3 (1, 8, 14, 23) MA242 TH 335-525 J COOK

#1 Meaningful / Meaningless

a) $(a \cdot b) \cdot c \Rightarrow$ less b/c dot product needs two vectors

b) $(a \cdot b)c \Rightarrow$ ful b/c $= \langle (a \cdot b)c_1, (a \cdot b)c_2, (a \cdot b)c_3 \rangle$

c) $|a|(b \cdot c) \Rightarrow$ ful b/c $= (\sqrt{a_1^2 + a_2^2 + a_3^2})(b_1c_1 + b_2c_2 + b_3c_3) \in \mathbb{R}$

d) $a \cdot (b + c) \Rightarrow$ ful b/c $= a \cdot b + a \cdot c$

e) $a \cdot b + c \Rightarrow$ less b/c can't add scaler to vector

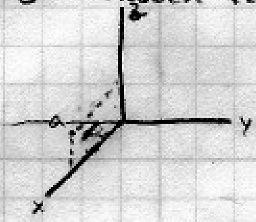
f) $|a| \cdot (b + c) \Rightarrow$ less b/c $|a|$ is scalar

#8 Find $a \cdot b$

$a = 4j - 3k$, $b = 2i + 4j + 6k$

$a \cdot b = [0(2) + 4(4) + -3(6)] = -2$

#14 Angle between vectors $a = \langle 4, 0, 2 \rangle$ $b = \langle 2, -1, 0 \rangle$



$|a| = \sqrt{4^2 + 0^2 + 2^2} = \sqrt{20}$

$|b| = \sqrt{2^2 + 1^2 + 0^2}$ $\sqrt{5}$

$a \cdot b = 4(2) + -1(0) + 2(0) = 8$

$\cos\theta = a \cdot b / |a||b| \Rightarrow 8 / (\sqrt{20})(\sqrt{5}) = .8 = \cos\theta$

$\cos^{-1}(.8) =$ $36.87°$

#23 Scalar & vector projections of b onto a

$a = \langle 3, -4 \rangle$ $b = \langle 5, 0 \rangle$

$|a| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ b onto a $= \text{Comp}_a b = a \cdot b / |a|$

$= (3)(5) + (-4)(0) / 5 = 3 = \text{Comp}_a b$

$\text{Proj}_a b = 3(a / |a|)$

$= 3 \langle 3, -4 \rangle / 5$

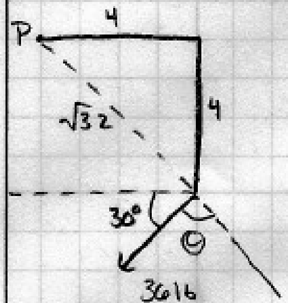
$= 3/5(3), 3/5(-4) = \langle 9/5, -12/5 \rangle$

9.4 (1, 6, 8, 22, 33) | MA242 TH 335-525 JCOOK

$\boxed{\#1}$ Meaningless / Meaningful

a) $a\cdot(b\times c) \Rightarrow$ ful $\rightarrow$ Scalar
b) $a\times(b\cdot c) \Rightarrow$ less
c) $a\times(b\times c) \Rightarrow$ ful $\rightarrow$ vector
d) $(a\cdot b)\times c \Rightarrow$ less
e) $(a\cdot b)\times(c\cdot d) \Rightarrow$ less
f) $(a\times b)\cdot(c\times d) \Rightarrow$ ful $\rightarrow$ scalar

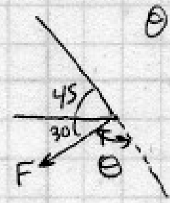
$\boxed{\#6}$



$|\tau| = |r\times F| = |r||F|\sin\cancel{30°}|n|$

$= (\sqrt{32})(36)\sin\cancel{30°}$
$= 203.65\sin\cancel{30°}$
≈ 101.8 lb

$\leftarrow$ wrong angle



$\theta = 180-30-45 = 105$

then sin 105 replaces sin 30 yielding

$\boxed{|\tau| = 197 \text{ ft lbs}}$

$\boxed{\#8}$ Find $a\times b$ & verify that it is orthogonal to both a and b

$a=\langle 5,1,4\rangle \quad b=\langle -1,0,2\rangle$

$$a\times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & 4 \\ -1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix}\hat{i} - \begin{vmatrix} 5 & 4 \\ -1 & 2 \end{vmatrix}\hat{j} + \begin{vmatrix} 5 & 1 \\ -1 & 0 \end{vmatrix}\hat{k}$$

$$= (2-0)\hat{i} - (10+4)\hat{j} + (0+1)\hat{k}$$

$$a\times b = 2\hat{i} - 14\hat{j} + \hat{k}$$

$\boxed{\#22}$ Find volume

$a = i+j-k \quad b = i-j+k \quad c = -i+j+k \quad = \langle 1,1,-1\rangle \ \langle 1,-1,1\rangle \ \langle -1,1,1\rangle$

$V = |a\cdot(b\times c)|$

$$b\times c = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} i - \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} j - \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} k$$

$$= (-1-1)i - (1+1)j - (1-1)k = -2i-2j = \langle -2,-2,0\rangle$$

$$a\cdot(b\times c) = \langle 1,1,-1\rangle\cdot\langle -2,-2,0\rangle = -4$$

$\cancel{|\langle -2,-2,0\rangle| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2} = V}$

$V = |a\cdot(b\times c)| = |-4| = \boxed{4}$

[mistake was to not realize $|\cdot|$ is absolute value in this context.]

$\boxed{\#33}$ $a \neq 0$

a) $a\cdot b = a\cdot c$ does $b=c$ $\Rightarrow$ No
b) $a\times b = a\times c$ does $b=c$ $\Rightarrow$ No
c) $a\cdot b = a\cdot c$ and $a\times b = a\times c$ does $b=c$ $\Rightarrow$ Yes

true but we should prove these statements!

$\boxed{\S 9.4 \# 33}$ Suppose $a \neq 0$.

(a.) $a \cdot b = a \cdot c \not\Rightarrow b = c$. For example $a = \hat{i}$, $b = \hat{j}$, $c = \hat{k}$ this is a counter-example $a \cdot b = a \cdot c = 0$ and $b \neq c$.

(b.) $a \times b = a \times c \not\Rightarrow b = c$. A counterexample would be $\hat{i} = a$ and $b = 2\hat{i}$, $c = 3\hat{i}$ then

$$a \times b = 2\hat{i} \times \hat{i} = 0 \text{ and } a \times c = 3\hat{i} \times \hat{i} = 0, \text{ yet } b \neq c.$$

(c.) Suppose $a \cdot b = a \cdot c$ and $a \times b = a \times c$. Intuitively we ought not be able to escape as in my counterexamples for case (a.) & (b.). Lets prove its true.

$$a \cdot b = a \cdot c \Rightarrow a \cdot (b - c) = 0 \quad \therefore a \perp (b - c)$$

$$a \times b = a \times c \Rightarrow a \times (b - c) = 0 \quad \therefore a \parallel (b - c).$$

Therefore $b - c = 0 \quad \therefore \boxed{b = c}$.

Sec 9.5

2) $r_0 = \langle 1, 0, -3\rangle = i + 0j - 3K \qquad V = 2i - 4j + 5K$

$r = (i + 0j + 3K) + t(2i - 4j + 5K)$

Vector equation $r = (1+t)i + (0 - 4t)j + (-3 + 5t)K$

parameteric equations are

$x = 1+t \qquad y = 0 - 4t \qquad z = -3 + 5t$

$\therefore\ x = 1+t \qquad y = -4t \qquad z = -3 + 5t.$

8) $r_0 = \langle 2, 1, 0\rangle = 2i + j + 0K$

$V = \begin{vmatrix} i & j & K \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} i - j\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + K\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$(1-0)i - j(1-0) + K(1-0)$

$r_0 + tV$

$V = i - j + K$

$r = (2i + j + 0K) + t(i - j + K)$

$r = (2+t)i + (1-t)j + (0+t)K.$

parameteric equations are

$x = 2+t \qquad y = 1-t \qquad z = 0+t$

$\therefore\ x = 2+t \qquad y = 1-t \qquad z = t$

Symmetric equation

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

$$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-0}{1}$$

SECTION 9.5

11) $(-4,-6,1)$ $(-2,0,-3)$

$V_1 = \langle -2-(-4),\ 0-(-6),\ -3-1 \rangle$
$= \langle 2, 6, -4 \rangle$

V_2 $(10, 18, 4)$ $(5, 3, 14)$

$V_2 = \langle 5-10,\ 3-18,\ 14-4 \rangle$
$\langle -5, -15, 10 \rangle$

$V_1 \times V_2 = 0$

$$= \begin{vmatrix} i & j & k \\ 2 & 6 & -4 \\ -5 & -15 & 10 \end{vmatrix}$$

alternatively notice
$V_2 = -\frac{5}{2} V_1$
thus $V_2 \parallel V_1$.

$$i\begin{vmatrix} 6 & -4 \\ -15 & 10 \end{vmatrix} - j\begin{vmatrix} 2 & -4 \\ -5 & 10 \end{vmatrix} + k\begin{vmatrix} 2 & 6 \\ -5 & -15 \end{vmatrix}$$

$$i(\underbrace{60-60}_{0}) - j(\underbrace{20-20}_{0}) + k(\underbrace{-30+30}_{0})$$

Therefore $V_1 \times V_2 = 0$ Yes both lines are parallel to each other

22) pt $(4,0,-3)$

vector $j + 2k = \langle 0, 1, 2 \rangle$

Putting $a = 0, b = 1, c = 2, x_0 = 4, y_0 = 0, z_0 = -3$

$0(x-4) + 1(y-0) + 2(z-(-3))$

$0 + y + 2z + 6 = 0$

$y + 2z = -6$

$\boxed{\S 9.5\#24}$ Find plane containing $r(t) = \langle 3+2t, t, 8-t \rangle$ and is parallel to $2x + 4y + 8z = 17$. We may choose $n = \langle 2, 4, 8 \rangle$ that will be parallel to $\langle 2, 4, 8 \rangle$. And a point on the plane is $r(0) = \langle 3, 0, 8 \rangle$ thus

$$2(x-3) + 4(y-0) + 8(z-8) = 0$$

$$\boxed{2x + 4y + 8z = 70}$$

$\boxed{\S 9.5\#32}$ Where does the line through $(1,0,1)$ and $(4,-2,2)$ intersect the plane $x + y + z = 6$. The line is

$$r(t) = (1,0,1) + t\,(4-1, -2-0, 2-1) = (x, y, z)$$

$$\left.\begin{aligned} x &= 1 + 3t \\ y &= -2t \\ z &= 1 + t \end{aligned}\right\} \text{ parametric eq}^{\text{n}}\text{'s of the line.}$$

The intersection with the plane of this line has $x+y+z=6$ simply substitute,

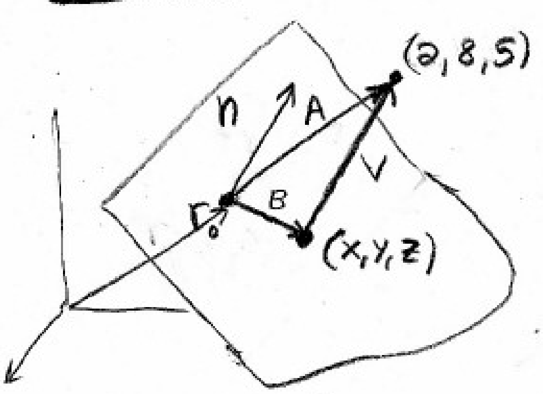
$$x+y+z = 1+3t - 2t + 1 + t = 6$$

$$2t = 6 - 2 = 4$$

$$\therefore t = 2 \Rightarrow r(2) \text{ is on the plane}$$

$$\boxed{r(2) = (7, -4, 3)}$$

$\boxed{\S 9.5\#47}$ Let $x - 2y - 2z = 1$ describe a plane. Find the distance to $(2,8,5)$.



$r_o = (0, 0, -1/2)$

$-2z = 1 \therefore z = -1/2$

$$V = (2-x, 8-y, 5-z)$$

$$n = \langle 1, -2, -2 \rangle$$

$$A = \langle 2, 8, 5.5 \rangle$$

$$\pm |V| = \text{comp}_n(A) = A \cdot \hat{n}$$

$$= \frac{1}{3} \langle 2, 8, 5.5 \rangle \cdot \langle 1, -2, -2 \rangle$$

$$= \frac{1}{3}(2 - 16 - 11)$$

$$= \frac{-25}{3} \Rightarrow \boxed{\text{distance is } \frac{25}{3}}$$

(the minus indicates its on the side opp. to n)