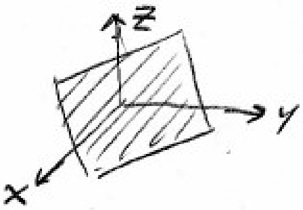
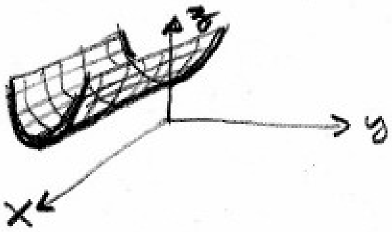
§9.6#6 See E18 from (257).

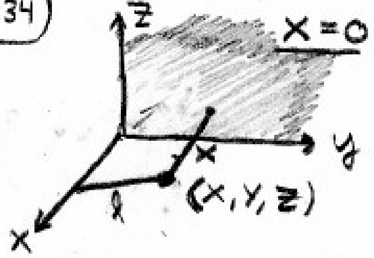
§9.6#11 This is a plane with normal $\langle -3, -2, -1 \rangle$ since
$z = f(x,y) = 6 - 3x - 2y \Rightarrow -6 = -3x - 2y - z$.



§9.6#13 $z = f(x,y) = y^2 + 1$, gives a parabola $z = y^2 + 1$ in each $x = x_o$ slice.



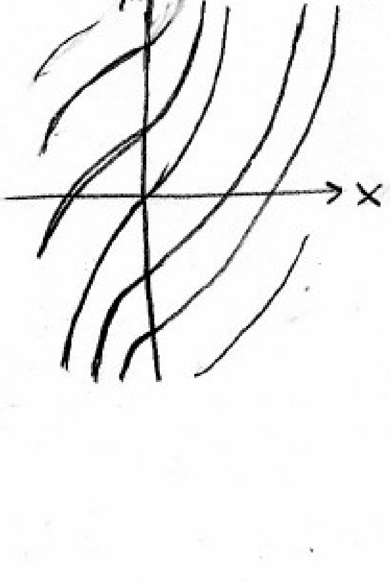
§9.6#34



$2\ell = x$ (from problem statement)
$\Rightarrow 4\ell^2 = x^2$
$\Rightarrow \boxed{4(y^2 + z^2) = x^2}$ this is a cone centered on x-axis.

§11.1#10 I. is a paraboloid since the level curves get denser as we get further from origin, this corresponds to $z = f(x,y)$ rising quickly. Plot II. is a cone because it has evenly spaced contours indicating a constant rate of increase as we travel away from the origin, in other words it has linear sides like a cone.

§11.1#16 $f(x,y) = x^3 - y$ plot a few level curves. These are $c = x^3 - y$ that is cubics $y = c + x^3$, I attempt a plot,



§11.1 #37 The level surfaces are $f(x,y,z) = \boxed{x + 3y + 5z = C}$ these are planes with normal $\langle 1, 3, 5\rangle$.

§11.1 #38 The level surfaces are $f(x,y,z) = \boxed{x^2 + 3y^2 + 5z^2 = C}$ these are ellipsoids.

§10.1 #4

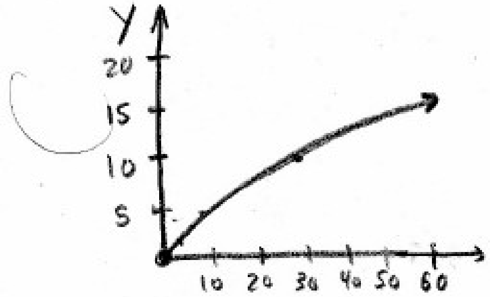
$$\lim_{t\to\infty} \langle \tan^{-1}(t), e^{-2t}, \tfrac{1}{t}\ln(t)\rangle = \langle \lim_{t\to\infty} \tan^{-1}(t), \lim_{t\to\infty}(e^{-2t}), \lim_{t\to\infty}\left(\frac{\ln(t)}{t}\right)\rangle$$

$$= \langle \pi/2, 0, \lim_{t\to\infty}\left(\frac{1/t}{1}\right)\rangle$$ using l'Hopital's on the 3rd limit.

$$= \boxed{\langle \pi/2, 0, 0\rangle}$$

§10.1 #6 Let $r(t) = \langle t^3, t^2\rangle$

t	x	y
0	0	0
1	1	1
2	8	4
3	27	9
4	64	16



$x = t^3 \Rightarrow x^2 = t^6$

$y = t^2 \Rightarrow y^3 = t^6$

$\therefore\ y^3 = x^2$

$\therefore\ y = x^{2/3}$

(CARTESIAN EQ^n for $r(t)$)

§10.1 #14 Find line from $P = (1,0,1)$ to $Q = (2,3,1)$

$r(t) = P + t(Q-P) = (1,0,1) + t(1,3,0) = \boxed{(1+t, 3t, 1) = r(t)}$

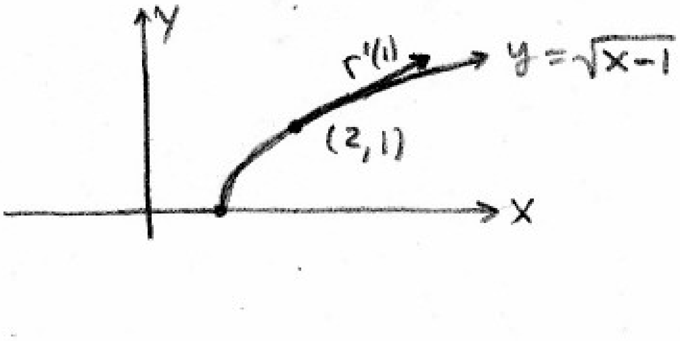
which means the parametric eq^n's are $x = 1+t$, $y = 3t$, $z = 1$.

§10.1 #38 See H19, sol^n is there.

§10.2 #4 $r(t) = \langle 1+t, \sqrt{t}\rangle$ $\quad r(1) = \langle 2, 1\rangle$

$r'(t) = \langle 1, 1/2\sqrt{t}\rangle$ $\quad r'(1) = \langle 1, 1/2\rangle$

So $x = 1+t$ and $y = \sqrt{t} = \sqrt{x-1}$



$\boxed{\S 10.2 \# 8}$ $r(t) = (1+\cos t)\hat{\imath} + (2+\sin t)\hat{\jmath}$

$r'(t) = (-\sin t)\hat{\imath} + (\cos t)\hat{\jmath}$

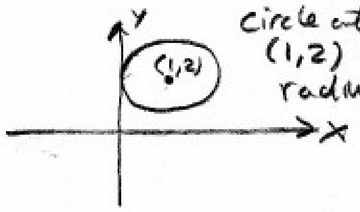
Thus $r(\pi/6) = (1+\sqrt{3}/2)\hat{\imath} + (3/2)\hat{\jmath}$ & $r'(\frac{\pi}{6}) = (\frac{-1}{2})\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}$

$l(t) = r(\pi/6) + t r'(\pi/6)$ ← standard method to find tangent line. just like $l(x) = f(a) + f'(a)(x-a)$ in the one-dim'l case.

$\therefore \boxed{l(t) = \langle 1+\frac{\sqrt{3}}{2} - \frac{1}{2}t,\ \frac{3}{2} + \frac{\sqrt{3}}{2}t \rangle}$

You could plot parametrically. Let me show what it is algebraically. Our goal is to eliminate t to obtain the Cartesian $Eq^{\underline{c}}$ of $r(t)$

$$x = 1+\cos t \qquad \therefore \quad \cos t = x-1$$
$$y = 2+\sin t \qquad\qquad \sin t = y-2$$



Circle at (1,2) with radius 1.

$$\cos^2 t + \sin^2 t = 1 = (x-1)^2 + (y-2)^2$$

$\boxed{\S 10.2 \# 12}$ $r(t) = \langle at\cos 3t,\ b\sin^3 t,\ c\cos^3 t \rangle$ where a, b, c are constants. Calculate the derivative with respect to t.

$$\frac{dr}{dt} = \langle \frac{d}{dt}(at\cos 3t),\ \frac{d}{dt}(b\sin^3 t),\ \frac{d}{dt}(c\cos^3 t) \rangle$$
$$= \langle a\cos 3t - 3at\sin(3t),\ 3b\sin^2 t\cos t,\ -3c\cos^2 t\sin t \rangle$$
$$= \boxed{\langle a(\cos 3t - 3t\sin(3t)),\ 3b\sin^2 t\cos t,\ -3c\cos^2 t\sin t \rangle = r'(t)}$$

$\boxed{\S 10.2 \# 14}$ Let $r(t) = ta \times (b+tc)$ where a, b, c are constant <u>vectors</u>

$$\frac{dr}{dt} = \frac{d}{dt}\left[ta \times (b+tc)\right]$$
$$= \frac{d}{dt}[ta] \times (b+tc) + ta \times \frac{d}{dt}[b+tc]$$: product rule for cross product.
$$= a \times (b+tc) + ta \times c$$: since $\frac{d}{dt}[ta] = a\frac{dt}{dt} = a$
$$= a \times (b+tc) + a \times (tc)$$ as $\frac{da}{dt} = 0$. And
$$= \boxed{a \times (b+2tc)}$$ also $\frac{db}{dt} = 0$ and $\frac{d}{dt}(tc) = \frac{dt}{dt}c = c$.

I like this problem.

$\boxed{\S 10.2 \# 15}$ $r(t) = \langle \cos t,\ 3t,\ 2\sin 2t \rangle$ find $T(t)$ and $T(0)$.

$$r'(t) = \langle -\sin t,\ 3,\ 4\cos(2t) \rangle$$

$$|r'(t)| = \sqrt{\sin^2 t + 9 + 16\cos^2(2t)}$$

$$T(t) = \frac{1}{|r'(t)|} r'(t) = \boxed{\frac{1}{\sqrt{\sin^2 t + 9 + 16\cos^2(2t)}} \langle -\sin t,\ 3,\ 4\cos(2t) \rangle}.$$

$$\therefore\ T(0) = \frac{1}{\sqrt{9+16}} \langle 0, 3, 4 \rangle = \boxed{\frac{1}{5}\langle 0, 3, 4 \rangle = T(0)}$$

$\boxed{\S 10.2 \# 28}$ Consider $r_1(t) = \langle t,\ 1-t,\ 3+t^2 \rangle$ and $r_2(s) = \langle 3-s,\ s-2,\ s^2 \rangle$ find where these intersect and find the angle of their intersection.

$$r_1(t) = r_2(s) \Rightarrow \begin{cases} t = 3-s \\ 1-t = s-2 \\ 3+t^2 = s^2 \end{cases} \Rightarrow \begin{aligned} &3+(3-s)^2 = s^2 \\ &3+9-6s+s^2 = s^2 \\ &\therefore\ 12 = 6s \quad \therefore\ \underline{s=2}\ \&\ \underline{t=1} \end{aligned}$$

Thus the point of intersection is $r_1(1) = r_2(2) = \langle 1, 0, 4 \rangle$. Next find the derivatives, the angle between $r_1'(1)$ and $r_2'(2)$ is what we interpret to be the angle between the curves.

$$r_1'(t) = \langle 1, -1, 2t \rangle \Rightarrow r_1'(1) = \langle 1, -1, 2 \rangle,\ |r_1'(1)| = \sqrt{6}$$

$$r_2'(s) = \langle -1, 1, 2s \rangle \Rightarrow r_2'(2) = \langle -1, 1, 4 \rangle,\ |r_2'(2)| = \sqrt{18}$$

$$r_1'(t) \cdot r_2'(s) = -1-1+8 = 6 = (\sqrt{6})(\sqrt{18})\cos\theta = 6\sqrt{3}\cos\theta$$

$$\therefore\ \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \cong \boxed{55^\circ = \theta}$$

$\boxed{\S 10.2 \# 29}$

$$\int_0^1 \langle 16t^3, -9t^2, 25t^4 \rangle dt = \left\langle \int_0^1 16t^3 dt,\ \int_0^1 -9t^2 dt,\ \int_0^1 25t^4 dt \right\rangle$$

$$= \left\langle 4t^4\Big|_0^1,\ -3t^3\Big|_0^1,\ 5t^5\Big|_0^1 \right\rangle = \boxed{\langle 4, -3, 5 \rangle}$$