$\boxed{\S 10.3\#1}$ $r(t) = \langle 2\sin t, 5t, 2\cos t \rangle$: $\underline{-10 \le t \le 10}$

$$r'(t) = \langle 2\cos t, 5, -2\sin t \rangle$$

$$|r'(t)| = \sqrt{4\cos^2 t + 25 + 4\sin^2 t} = \sqrt{29}$$

$$L = \int_{-10}^{10} |r'(t)|\,dt = \int_{-10}^{10} \sqrt{29}\,dt = \sqrt{29}\,t\Big|_{-10}^{10} = \boxed{20\sqrt{29} = L}$$

Remark: I should have always used L for the total arclength. We use s for the length of the arc from $r(a)$ upto $r(t)$.

$\boxed{\S 10.3\#8}$ $r(t) = \langle e^{2t}\cos 2t, 2, e^{2t}\sin 2t \rangle$ $t \ge 0$. $(a=0)$

$$r'(t) = \langle 2e^{2t}\cos 2t - 2e^{2t}\sin 2t, 0, 2e^{2t}\sin 2t + 2e^{2t}\cos(2t) \rangle$$

$$r'(t) = 2e^{2t}\langle \cos 2t - \sin 2t, 0, \sin 2t + \cos 2t \rangle$$

$$r'(t)\cdot r'(t) = \left[(\cos(2t) - \sin(2t))^2 + (\sin 2t + \cos 2t)^2\right]4e^{4t}$$

$$= [\cos^2 2t - \cancel{2\sin 2t\cos 2t} + \sin^2 2t$$

$$+\sin^2 2t + \cancel{2\sin 2t\cos 2t} + \cos^2 2t]4e^{4t}$$

$$= 8e^{4t} \quad \therefore \quad |r'(t)| = 2\sqrt{2}\,e^{2t}$$

$$s = \int_0^t |r'(u)|\,du = \int_0^t 2\sqrt{2}\,e^{2u}\,du = \sqrt{2}\,e^{2u}\Big|_0^t = \sqrt{2}(e^{2t} - 1) = s$$

$$\Rightarrow \quad s/\sqrt{2} + 1 = e^{2t} \quad \Rightarrow \quad t = \frac{1}{2}\ln\left(1 + s/\sqrt{2}\right)$$

$$r(t(s)) = \langle e^{\ln(1+s/\sqrt{2})}\cos(\ln(1+s/\sqrt{2})), 2, e^{\ln(1+s/\sqrt{2})}\sin(\ln(1+s/\sqrt{2}))\rangle$$

$$\boxed{r(s) = \langle (1+s/\sqrt{2})\cos(\ln(1+s/\sqrt{2})), 2, (1+s/\sqrt{2})\sin(\ln(1+s/\sqrt{2}))\rangle}$$

$\boxed{\S 10.3 \# 14}$ $r(t) = \langle t, \tfrac{1}{2}t^2, t^2 \rangle$

$r'(t) = \langle 1, t, 2t \rangle \Rightarrow |r'(t)| = \sqrt{1+5t^2}$

$$\boxed{T(t) = \frac{1}{\sqrt{1+5t^2}}\langle 1, t, 2t \rangle}$$

$$T'(t) = \frac{d}{dt}\left(\frac{1}{\sqrt{1+5t^2}}\right)\langle 1, t, 2t\rangle + \frac{1}{\sqrt{1+5t^2}}\frac{d}{dt}\left(\langle 1, t, 2t\rangle\right)$$

$$= -\tfrac{1}{2}(1+5t^2)^{-3/2}(10t)\langle 1, t, 2t\rangle + \frac{1}{\sqrt{1+5t^2}}\langle 0, 1, 2\rangle$$

$$= \frac{1}{(\sqrt{1+5t^2})^3}\left(-5t\langle 1, t, 2t\rangle + (1+5t^2)\langle 0, 1, 2\rangle\right)$$

$$= \frac{1}{(\sqrt{1+5t^2})^3}\langle -5t, -5t^2+1+5t^2, -10t^2+2+10t^2\rangle$$

$$\underline{= \frac{1}{(1+5t^2)^{3/2}}\langle -5t, 1, 2\rangle = T'(t)}$$

Now $|T'(t)| = (1+5t^2)^{-3/2}\sqrt{25t^2+5} = (1+5t^2)^{-3/2}(1+5t^2)^{1/2}\sqrt{5}$
thus $|T'(t)| = \sqrt{5}(1+5t^2)^{-1}$. Calculate $N(t)$ then,

$$N(t) \equiv \frac{1}{|T'(t)|}T'(t) = \frac{1+5t^2}{\sqrt{5}}\frac{1}{(1+5t^2)^{3/2}}\langle -5t, 1, 2\rangle$$

$$\boxed{N(t) = \frac{1}{\sqrt{5}\sqrt{1+5t^2}}\langle -5t, 1, 2\rangle}$$

$$\kappa(t) = \frac{|T'(t)|}{|r'(t)|} = \frac{\sqrt{5}(1+5t^2)^{-1}}{\sqrt{1+5t^2}} = \boxed{\frac{\sqrt{5}}{(1+5t^2)^{3/2}} = \kappa(t)}$$

§10.3 #17 I ignore Thm 10 and calculate directly as before. Find the curvature. Need $|T'(t)|$ and $|r'(t)|$.

$$r(t) = \langle 3t, 4\sin t, 4\cos t \rangle$$

$$r'(t) = \langle 3, 4\cos t, -4\sin t \rangle \quad \therefore \boxed{|r'(t)| = \sqrt{9+16} = 5}$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{1}{5}\langle 3, 4\cos t, -4\sin t \rangle$$

$$T'(t) = \frac{1}{5}\langle 0, -4\sin t, -4\cos t \rangle \quad \therefore \underline{|T'(t)| = \frac{1}{5}\sqrt{16} = 4/5}$$

$$\kappa(t) = \frac{|T'(t)|}{|r'(t)|} = \frac{4/5}{5} = \boxed{\frac{4}{25}} = \frac{a}{\sqrt{a^2+b^2}} \quad a = 4,\ b = 3$$

this is a helix afterall

§10.3#37 I leave to you, but the calculations are pretty much the same as the in-class lecture examples and other problems in this section.

§10.3#48 See the recommended hwk. sol^{n}'s.

§10.4#10

$$r(t) = \langle 2\cos t, 3t, 2\sin t \rangle$$

$$r'(t) = \boxed{\langle -2\sin t, 3, 2\cos t \rangle = v(t)}$$

$$|r'(t)| = \sqrt{4\sin^2 t + 9 + 4\cos^2 t} = \boxed{\sqrt{13} = \frac{ds}{dt}}$$

$$r''(t) = \boxed{\langle -2\cos t, 0, -2\sin t \rangle = a(t)}$$

§10.4#13

$$a(t) = \langle 1, 2, 0 \rangle$$

Remark: Compare this to the method and notation used in class. Both are correct.

$$v(t) = \int \langle 1,2,0 \rangle\, dt = \langle t, 2t, 0 \rangle + C_1$$

$$v(0) = \langle 0,0,1 \rangle = \langle 0,0,0 \rangle + C_1 \quad \therefore C_1 = \langle 0,0,1 \rangle$$

$$v(t) = \langle t, 2t, 1 \rangle \Rightarrow x(t) = \langle \tfrac{1}{2}t^2, t^2, t \rangle + C_2$$

$$x(0) = \langle 1,0,0 \rangle = \langle 0,0,0 \rangle + C_2 \Rightarrow \boxed{x(t) = \langle 1 + \tfrac{1}{2}t^2, t^2, t \rangle}$$

$\boxed{\S 10.4 \# 17}$ Given $r(t) = \langle t^2, 5t, t^2 - 16t \rangle$ find when speed is minimized.

$$r'(t) = \langle 2t, 5, 2t - 16 \rangle$$

$$|r'(t)| = \sqrt{4t^2 + 25 + (2t - 16)^2} = \frac{ds}{dt}$$

$$\frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{1}{2\sqrt{4t^2 + 25 + (2t-16)^2}} \left(8t + 2(2t-16)2\right) = 0$$

$$\frac{1}{\sqrt{\sim\sim}} \neq 0 \Rightarrow 12t - 64 = 0 \Rightarrow t = \frac{64}{12} = \underline{\frac{16}{3} = t}$$

observe $\frac{d}{dt}\left(\frac{ds}{dt}\right) > 0$ for $t > 16/3$ and $\frac{d}{dt}\left(\frac{ds}{dt}\right) < 0$ for $t < 16/3$ thus by the 1st derivative test $t = 16/3$ yields min. speed.

$\boxed{\S 10.4 \# 20}$ Suppose particle moves with constant speed say $A = \frac{ds}{dt}$ then the $V(t) = r'(t)$ and $a(t) = r''(t)$ are orthogonal.

$$|r'(t)| = \frac{ds}{dt} = A \Rightarrow r'(t) \cdot r'(t) = A^2$$

$$\Rightarrow r''(t) \cdot r'(t) + r'(t) \cdot r''(t) = 0$$

$$\Rightarrow r''(t) \cdot r'(t) = \boxed{a(t) \cdot V(t) = 0}$$

- I like this problem.

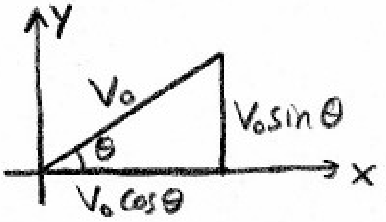
$\boxed{\S 10.4 \# 24}$ A gun has $\theta = 30°$, what is the needed muzzle velocity if the max-height of shell is 500m.

$$\underbrace{h_{max} = \frac{V_0^2 \sin^2 \theta}{2g}}_{\text{I prove on next page.}} = \frac{V_0^2}{8g} = 500$$

$$\therefore V_0 = \sqrt{4000g} = 20\sqrt{10}\, g$$

$$\boxed{V_0 \cong 198 \text{ m/s}}$$

$\boxed{\S 10.4 \# 24}$ $\quad a = \langle 0, -g \rangle$, $V(0) = \langle V_0 \cos\theta, V_0 \sin\theta \rangle$



$$a = \langle 0, -g \rangle$$

$$V = \langle 0, -gt \rangle + V(0).$$

$$V(t) = \langle V_0 \cos\theta, V_0 \sin\theta - gt \rangle$$

$$r(t) = \langle X_0 + V_0 \cos\theta\, t, Y_0 + V_0 \sin\theta\, t - \tfrac{1}{2} g t^2 \rangle$$

where $r(0) = \langle X_0, Y_0 \rangle$ as you can check. So

$$y = y_0 + V_0 \sin\theta\, t - \tfrac{1}{2} g t^2$$

$$\frac{dy}{dt} = V_0 \sin\theta - gt = 0 \therefore t = \frac{V_0 \sin\theta}{g}$$

(critical point)

$$\frac{d^2y}{dt^2} = -g < 0 \therefore \text{ its a max by } 2^{nd} \text{ der. test.}$$

$$y\left(\frac{V_0 \sin\theta}{g}\right) = y_0 + V_0 \sin\theta \left(\frac{V_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{V_0 \sin\theta}{g}\right)^2$$

$$h_{max} = y_0 + \frac{V_0^2 \sin^2\theta}{g}\left(1 - \frac{1}{2}\right)$$

Let $y_0 = 0$ then $\boxed{h_{max} = \dfrac{V_0 \sin^2\theta}{2g}}$

$\boxed{§10.4\#33}$ Suppose $r(t) = \langle \cos t, \sin t, t\rangle$. We could calculate T & N and find $a_T = r''(t) \bullet T$ and $a_N = r''(t) \bullet N$, and we will.

$$r'(t) = \langle -\sin t, \cos t, 1\rangle \quad \therefore |r'(t)| = \sqrt{2}$$
$$r''(t) = \langle -\cos t, -\sin t, 0\rangle$$

$$\boxed{T(t) = \frac{1}{\sqrt{2}}\langle -\sin t, \cos t, 1\rangle}$$

$$T'(t) = \frac{1}{\sqrt{2}}\langle -\cos t, -\sin t, 0\rangle \quad \therefore |T'(t)| = \frac{1}{\sqrt{2}}$$

$$\therefore N(t) = \frac{1}{T'(t)}\frac{dT}{dt} = \boxed{\langle -\cos t, -\sin t, 0\rangle = N(t)}$$

$$a_T = r''(t)\bullet T = \langle -\cos t, -\sin t, 0\rangle \bullet \frac{1}{\sqrt{2}}\langle -\sin t, \cos t, 1\rangle$$
$$= \frac{1}{\sqrt{2}}(\cancel{\cos t \sin t} - \cancel{\sin t \cos t} + 0) = \boxed{0 = a_T}$$

$$a_N = r''(t)\bullet N = \langle -\cos t, -\sin t, 0\rangle \bullet \langle -\cos t, -\sin t, 0\rangle$$
$$= \cos^2 t + \sin^2 t$$
$$= \boxed{1 = a_N}$$

$\boxed{§10.4\#36}$ $L(t) = m r(t) \times v(t)$

$$\frac{dL}{dt} = m\frac{d}{dt}(r\times v) = m\Big(\underbrace{\frac{dr}{dt}\times v}_{v\times v = 0} + r \times \frac{dv}{dt}\Big) = mr \times \frac{dv}{dt}$$

$$\therefore \frac{dL}{dt} = r\times\left(m\frac{dv}{dt}\right) = r\times F = \tau = \underline{\text{torque}}$$

time-rate of change of angular momentum