Summary of material for Test I: Sections 9.1-9.6, 10.1-10.3, 10.4

- Representation in 3D or R^3
- Distance between two points
- Finding a component vector between two points
- Operations on vectors (+, -, scalar multiple)
- Magnitude of a vector or norm of a vector
- Unit vectors
- Dot product and cross product of two vectors
- Angle between two vectors (or two planes)
- Orthogonal vectors
- Scalar and vector projections
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- Parametric equations of a line given sufficient information
- Able to indicate a direction vector that is parallel to the line when given parametric equations of the line
- Determine whether two given lines are parallel, intersecting or skew. If they intersect, find the point of intersection (skew means nonintersecting and not parallel.)
- Find equation of a plane when given sufficient information
- Determine the point where the line intersects a given plane
- Determine whether two given planes are parallel, perpendicular, or neither. If neither, determine the angle between them and determine the line of intersection of two given planes
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- Find the (unit) tangent vector or the derivative of a given vector function $r(t)$
- Find parametric equations for the tangent line to $r(t)$ at a given specific point (when given a point, you may have to determine the value of t)
- Integrals involving vectors functions
- Find the length of a space curve when given $r(t)$
- Given $r(t)$ find T, N, B, arclength function and curvature.

YOU DO NOT HAVE TO DRAW ANY THING ☺

- find speed, velocity, acceleration given $r(t)$ the position
- break up the acceleration into $a_T T + a_N N$, that is find a_T & a_N.
- read homework and lecture examples.

Likely Test Format:

PROBLEM ONE Given A, B find $\hat{A}, \hat{B}, A \cdot B, \Theta$, angle between some sum or difference of A, B and another. And find a $\perp$ vector to A, B.

PROBLEM TWO Eq^n of Plane given some data.

PROBLEM THREE Problem involving sphere intersecting something, general topic eq^n's and intersections how to use algebra to describe intersections.

PROBLEM FOUR Integrate some vector function of t.

PROBLEM FIVE Given $r(t)$ find speed, arclength function, T, N, B the curvature, acceleration, a_T and a_N

PROBLEM SIX Conceptual question involving vector addition, subtraction and components / projections.

Bonus (5pts) ← Some question involving Einstein Summation tricks I'll give the basics

← torsion for problem 5 and the osculating plane.

Formula Sheet for Test I, ma 242-011

SPHERE of radius R and center (h,k,l) is $(x-h)^2+(y-k)^2+(z-l)^2 = R^2$.

If $a = \langle a_1, a_2, a_3\rangle$ then $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

If P, Q are points then $P-Q$ is directed line segment from Q to P and the distance from Q to P is $d(P,Q) = |PQ| = |P-Q|$

$\langle a_1, a_2\rangle + \langle b_1, b_2\rangle = \langle a_1 + b_1, a_2 + b_2\rangle$

$c\langle a_1, a_2\rangle = \langle ca_1, ca_2\rangle$

If $a, b \neq 0$ then $a \cdot b = |a||b|\cos\theta$ for $0 \le \theta \le \pi$.

Also $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$ and $a \cdot b = 0 \Leftrightarrow a \perp b$.

$\text{comp}_a(b) = a \cdot \hat{b}$ the component of a in the b-direction.

$\text{proj}_a(b) = (a \cdot \hat{b})\hat{b}$ the vector component of a in the b-direction.

If $a, b \neq 0$ then $a \times b = |a||b| \sin\theta\, \hat{n}$ where $0 \le \theta \le \pi$ and $\hat{n}$ is given by right hand rule. Also for any vectors a, b

$$a \times b = \langle a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1\rangle = \begin{vmatrix} \vec{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

A line through r_0 with direction v can be written $r(t) = r_0 + tv$

A plane through r_0 with normal $n = \langle a, b, c\rangle$ is $n \cdot (r - r_0) = 0 = a(x-x_0) + b(y-y_0) + c(z-z_0)$.

If u, v are vector-valued functions of a real variable and f is a real valued fnct. and c is a constant then

$$\frac{d}{dt}[fu] = \frac{df}{dt}u + f\frac{du}{dt} \qquad \frac{d}{dt}[u \cdot v] = \frac{du}{dt} \cdot v + u \cdot \frac{dv}{dt}$$

$$\frac{d}{dt}[u \times v] = \frac{du}{dt} \times v + u \times \frac{dv}{dt} \qquad \frac{d}{dt}[u(f(t))] = f'(t)\, u'(f(t)) = \left.\frac{du}{dt}\right|_{f(t)} \frac{df}{dt}$$

For appropriate $r: [a,b] \to \mathbb{R}^3$ then $L = \int_a^b |r'(t)|\,dt$ the arclength function is $s = \int_a^t |r'(u)|\,du = \int_0^t \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2}\,du$

For appropriate $r(t)$,

$$T(t) = \frac{r'(t)}{|r'(t)|} \qquad N(t) = \frac{T'(t)}{|T'(t)|} \qquad B(t) = T(t) \times N(t)$$

$$\kappa = \left|\frac{dT}{ds}\right| = \frac{1}{|r'(t)|}|T'(t)|$$

$$\frac{dT}{ds} = \kappa N \qquad \frac{dN}{ds} = -\kappa T + \tau B \qquad \frac{dB}{ds} = -\tau N$$

T = tangent
N = normal
B = binormal
κ = curvature
τ = torsion

If $r(t)$ denotes the position at time t then

$v(t) = \frac{dr}{dt}$ = velocity, $a(t) = \frac{dv}{dt}$ = acceleration $|v(t)| = \frac{ds}{dt}$ = speed $= \dot{s}$

$$a = \ddot{s}\,T + \kappa(\dot{s})^2 N = a_T T + a_N N$$