Test III overview and study guide

Test III covers double and triple integrals (chapter 12) and curvelinear coordinate systems (section 9.7) [this covers most of pages 330-359]. As usual your first and best line of defense is to do the homework carefully. Next, it is important to understand all the lecture examples that I covered in lecture.

Cartesian Topics, sections 12.1, 12.2, 12.3, 12.7 [330-342 my notes]

- double integrals over rectangles (E87-E88) [hwk 12.2#6,10,12,14,16,23,31]
- triple integrals over boxes (E89-E90) [hwk 12.7#2 for example]
- double integrals over general regions (E93-E99) [hwk 12.3#3,10,12,15,19,33,40,43]
- triple integrals over general volumes (E100-E103) [hwk 12.7#5,6,8,10,14,19]
- be able to calculate bounded volumes. This was done by two methods. $V = \int\int_R f(x,y)dA$ verses $V = \int\int\int_B dV$, how would B and R be related if both represent the same volume ?
- be able to calculate the area of a region in the xy-plane.
- if another application besides volume/area appears on the test I will give you the formula so you just have to integrate.
- what is the difference between $\int\int_R f(x,y)dA$ and $\int_0^1\int_{x^2}^x f(x,y)dydx$? If these are equal then what is R? Graph R.
- what is the difference between $\int\int\int_B f(x,y,z)dV$ and $\int_0^1\int_0^1\int_0^{y+1} f(x,y,z)dzdydx$? If these are equal what is B? Sketch B.
- know the definitions of TYPE I and TYPE II. Be able to switch bounds when helpful (like E99).

Coordinate change and integration, sections 9.7, 12.9, 12.4, 12.8 [343-359 my notes]

Notice I covered these in an order different then Stewart. I cover 12.9 first so that we can derive the basic formulas we use in 12.4 (double integrals in polars) and 12.8(triple integrals in cylindricals and sphericals). Of course to understand 12.4 and 12.8 we must have the basics of 9.7 fresh in mind.

- I will give you the coordinate change theorem and definition of Jacobian on page 344.
- I will give you the definition of Jacobian on page 347 and the theorem on page 349.
- I will give you that $dA = rdrd\theta$ for polars, $dV = rdrd\theta dz$ for cylindricals and $dV = \rho^2 \sin\phi d\rho d\theta d\phi$ for sphericals. Generally,

$$dA = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv \qquad dV = \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| dudvdw.$$

  with these in mind we can construct the theorems, really its just another way of saying the coordinate change theorem. You must change the measure (dA or dV) and substitute everywhere the new variables for the old. Also we must change the bounds, this is just like u-substitution. The wrinkle that might get lost in this way of thinking is that we must order du, dv, dw in the way that makes sense. We must follow the common sense of iterated integrals, compicated bounds on the inside, numeric bounds on the outside. The Theorem on page 349 should be understood in that context.

- I may ask you to work through E109 or E110. Or other Jacobians as in 12.9# 1,4,6.
- I will give the defining equations for polars, cylindrical and sphericals. (for example $x = r\cos(\theta), y = r\sin(\theta)$, see section 9.7 or pages 349-350.)
- you should be able to convert a region(or volume) from its Cartesian description into its polar coordinate description (or cylindrical or spherical for volumes). This is essential if you are to successfully change integrals from Cartesians to other coordinates.
- double integrals in polar coordinates (E114-116) [hwk from 12.4 H78-H80]
- triple integrals in cylindrical coordinates (E117) [12.8#9,32]
- triple integrals in spherical coordinates (E118-E120)[12.8#18]
- given an integral in Cartesian coordinates be able to change coordinates wisely. That is have an idea about which coordinate system makes $x^2 + y^2 + z^2$ into something simple, or what about $x^2 + y^2$. [Answer: sphericals and cylindricals or polars respectively]. You also had a homework problem(12.9#21) involving this idea, if it was something sneaky like that I would suggest equations defining u, v and if appropriate w.

assumed background knowledge

Let me give you an (incomplete) list of the integrals I expect you to know. I have mentioned I will shy away from certain more fussy integrals, but you should certainly be able to do basic u-substitutions and elementary integrals. The "elementary integrals" are given on page 91 (or 98) of my lecture notes. Also you should be able to integrate

$$\sin(x), \quad \sin^2(x) \ \sin^3(x), \quad \sin^4(x), \quad \sin^5(x), \quad \cos(x),$$

$$\cos^2(x), \quad \cos^3(x), \quad \cos^4(x), \quad \cos^5(x), \quad \sin(x)\cos(x), \quad \cos(x)\sin^2(x), \quad \cos^2(x)\sin^2(x)$$

and so on... I'll supply the needed trig identities if you ask me, see pages 105-106 for details on how to do some of these. Most of the integrals will be pretty simple, I'm just trying to alert you to some of the strange ones that come up when changing to polars, cylindricals or sphericals.