TABLE 7.1 BRIEF TABLE OF LAPLACE TRANSFORMS

$f(t)$	$F(s) = \mathscr{L}\{f\}(s)$	
1	$\dfrac{1}{s}$,	$s > 0$
e^{at}	$\dfrac{1}{s-a}$,	$s > a$
t^n, $n = 1, 2, \ldots$	$\dfrac{n!}{s^{n+1}}$,	$s > 0$
$\sin bt$	$\dfrac{b}{s^2+b^2}$,	$s > 0$
$\cos bt$	$\dfrac{s}{s^2+b^2}$,	$s > 0$
$e^{at}t^n$, $n = 1, 2, \ldots$	$\dfrac{n!}{(s-a)^{n+1}}$,	$s > a$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$,	$s > a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$,	$s > a$

TABLE 7.2 PROPERTIES OF LAPLACE TRANSFORMS

$\mathscr{L}\{f+g\} = \mathscr{L}\{f\} + \mathscr{L}\{g\}$.

$\mathscr{L}\{cf\} = c\mathscr{L}\{f\}$ for any constant c.

$\mathscr{L}\{e^{at}f(t)\}(s) = \mathscr{L}\{f\}(s-a)$.

$\mathscr{L}\{f'\}(s) = s\mathscr{L}\{f\}(s) - f(0)$.

$\mathscr{L}\{f''\}(s) = s^2\mathscr{L}\{f\}(s) - sf(0) - f'(0)$.

$\mathscr{L}\{f^{(n)}\}(s) = s^n\mathscr{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$.

$\mathscr{L}\{t^nf(t)\}(s) = (-1)^n\dfrac{d^n}{ds^n}\left(\mathscr{L}\{f\}(s)\right)$.

(8) $\mathscr{L}\{g(t)u(t-a)\}(s) = e^{-as}\mathscr{L}\{g(t+a)\}(s)$.

$\mathscr{L}\{u(t-a)\}(s) = \frac{1}{s}e^{-as}$

$\mathscr{L}\{\delta(t-a)\}(s) = e^{-as}$

$\mathscr{L}\{t^n\}(s) = \dfrac{\Gamma(n+1)}{s^{n+1}}$

TRANSLATION IN t

Theorem 8. Let $F(s) = \mathscr{L}\{f\}(s)$ exist for $s > \alpha \geq 0$. If a is a positive constant, then

(5) $\mathscr{L}\{f(t-a)u(t-a)\}(s) = e^{-as}F(s)$,

and, conversely, an inverse Laplace transform† of $e^{-as}F(s)$ is given by

(6) $\mathscr{L}^{-1}\{e^{-as}F(s)\}(t) = f(t-a)u(t-a)$.

TRANSFORM OF PERIODIC FUNCTION

Theorem 9. If f has period T and is piecewise continuous on $[0, T]$, then

(12) $$\mathscr{L}\{f\}(s) = \frac{F_T(s)}{1-e^{-sT}} = \frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}} .$$

CONVOLUTION

Definition 8. Let $f(t)$ and $g(t)$ be piecewise continuous on $[0, \infty)$. The **convolution** of $f(t)$ and $g(t)$, denoted $f * g$, is defined by

(3) $$(f*g)(t) := \int_0^t f(t-v)g(v)\,dv .$$

CONVOLUTION THEOREM

Theorem 11. Let $f(t)$ and $g(t)$ be piecewise continuous on $[0, \infty)$ and of exponential order α and set $F(s) = \mathscr{L}\{f\}(s)$ and $G(s) = \mathscr{L}\{g\}(s)$. Then

(8) $\mathscr{L}\{f*g\}(s) = F(s)G(s)$,

or, equivalently,

(9) $\mathscr{L}^{-1}\{F(s)G(s)\}(t) = (f*g)(t)$.

← unless I say otherwise you need to complete the integrals implicit within $f*g$ for full credit