# Ma341-004: Final Exam

Friday, June 28, 2005
8:00 a.m. — 11:00 a.m.
Instructor: Dr. Bill Cook

- **Show all of your work.**
- **Do not write your answers or work on the exam.**

**#1 (10 points)**

(a) Solve the following initial value problem:

$$y' + \frac{1}{x}y = 2e^{x^2}, \qquad y(1) = 0.$$

(b) Give a general (implicit) solution for the following differential equation:

$$\left(3x^2y^2 + \frac{1}{x}\right)\,\mathrm{d}x + \left(2x^3y + \cos(y)\right)\,\mathrm{d}y = 0.$$

**#2 (15 points)**

(a) Solve the following initial value problem:

$$y'' + 4y' + 4y = 0, \qquad y(0) = 0, \qquad \text{and} \qquad y'(0) = 1.$$

(b) Solve the following differential equation:

$$y'' - 6y' + 13y = 30\sin(x) + 26x + 1.$$

**#3 (10 points)**

(a) Compute the Laplace transform of $f(t)$ where

$$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ t & 1 < t \end{cases}$$

(b) Compute the inverse Laplace transform of $F(s)$ where

$$F(s) = \frac{e^{-2s}}{(s-1)^2} + \frac{e^{-s}s}{s^2+9}$$

**#4 (15 points)** Use Laplace transforms to solve the following initial value problem:

$$y'' - 2y' + y = -4e^t, \qquad y(0) = 0, \qquad \text{and} \qquad y'(0) = 1.$$

**#5 (15 points)** Consider the following system of differential equations:

$$x'(t) = y'(t)$$

$$y''(t) = x(t) - y(t)$$

(a) Convert this system into an equivalent system of first order differential equations.

(b) Solve the original system (*note:* you can use part (a) or use the elmination method).

**#6 (15 points)** Let

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{f}(t) = \begin{bmatrix} 1 \\ e^t \end{bmatrix}.$$

(a) Compute the matrix exponential $e^{\mathbf{A}t}$.

(b) Find a general solution for $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{f}(t)$.

**#7 (15 points)** Consider the following system of autonomous differential equations:

$$\frac{dx}{dt} = -y + 1 \qquad \text{and} \qquad \frac{dy}{dt} = x - 1.$$

(a) Write down the phase plane equation and solve it.

(b) Graph several sample trajectories.

(c) What is the critial point set?

**#8 (10 points)**

Find the critical point for each system. Is it stable, asymptotically stable, or unstable? Is it a proper node, improper node, saddle point, spiral point, or center?

(a)

$$\begin{aligned} x' &= 2x + 3y \\ y' &= -y + 2 \end{aligned}$$

(b)

$$\begin{aligned} x' &= -x - 2y - 5 \\ y' &= 8x - y + 6 \end{aligned}$$