

Ma341-001: Test #1

Friday, June 3, 2005

Instructor: Dr. Bill Cook

- Show all of your work.
- Do not write your answers or work on the test.

#1 (15 points) Classify the following equations:

- (a) What kind of differential equation is " $y' = x - yx$ "?
separable, linear, neither, or both
- (b) What kind of differential equation is " $y y' = \ln(x + y)$ "?
separable, linear, neither, or both
- (c) What kind of differential equation is " $(e^{x+y} - \sin(x)) dx + (e^{x+y} + 2y) dy = 0$ "?
linear, exact, neither, or both

#2 (18 points) A 10 liter tank is initially filled with pure water. A brine with a concentration of 2 kilograms of salt per liter is pumped into the tank at a rate of 5 liters per minute. The brine in the tank is well mixed and drained at a rate of 5 liters per minute. Let $x(t)$ denote the amount of salt (in kilograms) in the tank after t minutes have gone by.

- (a) State an initial value problem which $x(t)$ satisfies.
- (b) Draw the phase line for the differential equation found in part (a).
- (c) What is $\lim_{t \rightarrow \infty} x(t)$?

#3 (32 points) Solve the following initial value problems:

(You do not need to make your solutions explicit. Implicit solutions are fine.)

- (a) $y \frac{dy}{dx} = \frac{\sec(y^2)}{x}, \quad y(1) = 0$
- (b) $\frac{dy}{dx} + \frac{y}{x} = e^{-x}, \quad y(1) = e^{-1}$
- (c) $(3x^2 + 2xy^2) dx + (2x^2y + 3) dy = 0, \quad y(0) = 2$
- (d) $y'' - y' - 2y = 0, \quad y(0) = 2 \text{ and } y'(0) = 1$

#4 (30 points) Find general solutions for the following equations:

- (a) $y'' + 6y' + 13y = 13x + 19$
- (b) $y'' + 2y' + y = 2e^{-t}$
- (c) $y'' + y = \csc(x)$

#5 (10 points) Find linear differential operators which annihilate the following functions:

- (a) $6xe^{2x} + 5$
- (b) $2e^{-x} \sin(3x) + 15e^{-x} \cos(3x) + 7e^{4x}$