# Ma341-004: Test #2

Friday, June 14, 2005
Instructor: Dr. Bill Cook

- **Show all of your work.**
- **Do not write your answers or work on the test.**

**#1 (24 points)** Compute the Laplace transform, $\mathcal{L}\{f(t)\}(s)$, for each of the following:

(a) $f(t) = \dfrac{1}{\sqrt{t}} + te^{-2t} - 1$ (*Hint:* $\Gamma(\frac{1}{2}) = \sqrt{\pi}$)

(b) $f(t) = e^{2t}\cos(3t) - 5e^{-t}\sin(2t)$

(c) $f(t) = t\sin(t)$

(d) $f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ t & 1 < t \le 2 \\ e^t - 2 & 2 < t \end{cases}$

**#2 (32 points)** Compute the inverse Laplace transform, $\mathcal{L}^{-1}\{F(s)\}(t)$, for each of the following:

(a) $F(s) = \dfrac{s+1}{s^2 - 2s + 5}$

(b) $F(s) = \dfrac{2s^2 + 3s - 1}{(s^2+1)(s-1)}$

(c) $F(s) = \dfrac{e^{-s}(4s+2)}{s(s+1)}$

(d) $F(s) = \dfrac{12s}{(s^2+4)(s^2+9)(s-1)s^4}$

*Note:* For part (d), please give your answer in terms of a (multi-)convolution product

**#3 (30 points)** Solve the following initial value problems using Laplace transforms.

(a) $y'' + y = 2e^t$, $y(0) = 1$, and $y'(0) = 2$.

(b) $y'' - 2y' + y = u(t-1)$, $y(0) = 0$, and $y'(0) = 1$ ($u(t)$ is the unit step function).

**#4 (16 points)** Use Laplace transforms to solve the following system of differential equations:

$$\begin{aligned} x' &= x + y & x(0) &= 0 \\ y' &= x + y & y(0) &= 6 \end{aligned}$$