Ma341-004: Test #3

Friday, June 22, 2005
Instructor: Dr. Bill Cook

- **Show all of your work.**

- **Do not write your answers or work on the test.**

#1 (15 points) Consider the following system of differential equations:

$$\begin{aligned} y'''(t) + 2y'(t) - x(t) &= 6 \\ x''(t) + y''(t) - 2x'(t) &= \sin(t) \end{aligned}$$

(a) Convert this system into an equivalent system of first order differential equations.

(b) Rewrite your answer to part (a) in matrix normal form.

#2 (20 points) Find the general solution for the following system of differential equations:

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix} \mathbf{x}(t).$$

#3 (20 points) Solve the following initial value problem:

$$\mathbf{x}'(t) = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \mathbf{x}(t) \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Please note:

The matrix $\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$ has an eigenvalue $-1 + i$ with corresponding eigenvector $\begin{bmatrix} i \\ 1 \end{bmatrix}$.

#4 (20 points) Compute the matrix exponential e^{At} where

$$A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}.$$

#5 (25 points) Find the general solution for the following system of differential equations:

$$\mathbf{x}'(t) = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ e^t \end{bmatrix}.$$

Please note:

$$X(t) = \begin{bmatrix} 1 & 2e^t \\ 0 & e^t \end{bmatrix} \text{ is a fundamental matrix for } \mathbf{x}'(t) = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \mathbf{x}(t).$$