Introduction to Differential Equations: Homework on Systems of ODEs

Directions: Show your work, if you doubt that you've shown enough detail then ask. My instructions supercede the intstructions in the text.

1. (10pts) Do problem 9.1 # 12. See page 84 E3 in my notes. Also solve 9.2 # 9.

2. (10pts) Find the inverse of X without technology,
$$X = \begin{pmatrix} \sin(2t) & \cos(2t) \\ 2\cos(2t) & 2sin(2t) \end{pmatrix}$$
Find the inverse of Y, you may use technology,
$$Y = \begin{pmatrix} e^{3t} & 1 & t \\ 3e^{3t} & 0 & 1 \\ 9e^{3t} & 0 & 0 \end{pmatrix}$$

3. (10pts) Do 9.4 # 26 and 28.

4. (10pts) Find the eigenvalues and eigenvectors of the matrix below
$$A = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}$$
then find the general solution to $\frac{d\vec{x}}{dt} = A\vec{x}$.

5. (10pts) Find the eigenvalues and eigenvectors of the matrix below
$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$
then find the general solution to $\frac{d\vec{x}}{dt} = A\vec{x}$.

6. (10pts) Complete 9.5 # 50.

7. (10pts) Complete 9.6 # 19. For 10 bonus points on the homework you may solve the system and animate the behaviour, I'd like 3 distict cases illustrated. I believe there is a way of saving an animated .gif file, if you look at my calc III page I have an example of the code for that.

8. (10pts) Use chapter 9 techniques to solve 5.2 # 38. (I would like you to use the eigenvalue/eigenvector machine plus variation of parameters)

9. (10pts) Find the eigenvalues and eigenvectors and generalized eigenvectors of the matrix below
$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$
then find the general solution to $\frac{d\vec{x}}{dt} = A\vec{x}$. You should use the ideas we developed concerning the matrix exponential to form the solution. The next problem may help with the logic needed here.

10. (10pts) Suppose that A is a 9x9 matrix such that $det(A - \lambda I) = (\lambda - 1)^2(\lambda - 3)^3(\lambda^2 + 1)^2$. Furthermore, suppose that

$$(A - I)\vec{u_1} = 0 \qquad and \qquad (A - I)\vec{u_2} = 0$$

where $\vec{u_1}, \vec{u_2}$ are nontrivial and linearly independent. Next suppose that,

$$(A - 3I)\vec{u_3} = 0 \qquad and \qquad (A - 3I)\vec{u_4} = \vec{u_3} \qquad and \qquad (A - 3I)\vec{u_5} = \vec{u_4}$$

where $\vec{u_3}, \vec{u_4}, \vec{u_5}$ are all nontrivial. Next suppose that $\vec{u_6} = \vec{a_6} + i\vec{b_6}$ is a complex eigenvector with,

$$(A - iI)\vec{u_6} = 0$$

where $\vec{a_6}, \vec{b_6}$ are real vectors. Finally suppose that $\vec{u_7} = \vec{a_7} + i\vec{b_7}$ is a generalized complex eigenvector with,

$$(A - iI)\vec{u_7} = \vec{u_6}$$

where $\vec{a_7}, \vec{b_7}$ are real vectors.

Given all this data calculate the general (real) solution to $\frac{d\vec{x}}{dt} = A\vec{x}$ in terms of the given vectors.