Differential Equations Test II Overview

As always your first best line of defense is to complete and understand the homework and lecture examples. I will rule out a few of the problems which were assigned for breadth, namely 9.3#14, 20, 9.5#24, 9.7#1, 4, 5, 7, 9, 9.8#10 (just the part about the inverse of the 3x3) Also please redo the generalized eigenvector problems using the chain, 9.7#17, in my posted solution I don't bother to use the chain, but it would be good practice. Some of the items below overlap, if I repeat something that probably means it is important. Also beware the numbers given below are taken from my homework solutions, they may not match up with the problems in the text (see the errata page for details). I have just pointed to the problems in the homework solution which are relevant to a given comment. There are additional homeworks available in your text if you need further practice.

1. Be able to convert a linear system of ODEs to a matrix ODE in normal form. Like 9.1#11, 9.4#3, 5, 7, 9.5#41.
2. Know how to do matrix math, multiply, add, matrix-product rule etc... See 9.3#3, 4 and most everything we do.
3. If A is a 2x2 matrix what is A^{-1}? When and how can you compute it easily ? See 9.3#18, 19. Please note that I said no 3x3 inverse on the test (unless I just give it to you) so you need not worry about calculating the inverse of a 3x3 by hand, can ignore 9.3#14, 20 they're not on test.
4. Be able to calculate the determinant of 2x2, 3x3 and 4x4 matrices.
5. What is a solution matrix for a system in matrix normal form ? See 9.3#37.
6. What is a fundamental matrix for $\vec{x}' = A\vec{x}$? See 9.4#26, 28 9.5#19(*see my solution*), 9.6#6(*see my solution*), 9.8#10.
7. What does the Wronskian tell us about linear indepence ? What is the Wronskian ? When and why might you calculate the Wronskian ?
8. Given two matrices of functions X and Y how would you try to see if they were solution matrices to the same system ? How would you propose to check if they were both fundamental matrices for some system (suppose we don't know A explicitly)? This is a challange question, you may go on to more fundamental items then come back to this later.
9. Be able to find eigenvalues.
10. Be able to calculate eigenvectors for real eigenvalues. See 9.5#1, 4, 6, 7, 9, 26, 31, 9.8#10.
11. Be able to calculate eigenvectors for complex eigenvalues. See 9.5#9, 9.8#2, 6, 13.
12. Be aware of the weird trick to calculate e^{At} using a fundamental matrix (see 9.8#1, 7). Also be aware of the special case where the matrix exponential is easy to calculate (see 9.8#5)
13. Be able to calculate generalized eigenvectors. Know how to use the chain condition to make life easier when we use the generalized eigenvector to extract the solution from the matrix exponential.
14. Understand the calculation I did on page 97. Be able to reproduce it so you can also extract the two real solutions within the complex solution.
15. Be able to solve $\vec{x}' = A\vec{x}$ using eigenvectors and generalized eigenvectors (if needed). I may ask an explicit generalized eigenvector question where you actually use the chain condition on an explicit matrix (in contrast to PROBLEM 10 in your homework). I mentioned numerous times that all of the solutions come from $\vec{x_k} = e^{At}\vec{u_k}$ where $\vec{u_k}$ is a generalized eigenvector of order k (of course usually we just use eigenvectors). How many linearly independent solutions do you need? Where and how will a t or a t^2 appear in our general solution?
16. Be able to solve $\vec{x}' = A\vec{x} + \vec{f}$ using variation of parameters. Be able to prove that $\vec{x_p} = X\int X^{-1}\vec{f}dt$ is a solution, also be able to use it for A a 2x2 matrix.
17. The homework is a little weak on explicit generalized eigenvectors, don't forget to look over the examples we covered in class, make sure you understand the chain idea.

18. Why do we use generalized eigenvectors ? Be able to prove that e^{At} is a fundamental matrix.

19. Be able to fit initial conditions to the general solution.

The test will be similar in content to my old test III's. I plan to give you the basic formula for the matrix exponential, however, it's upto you to interpret it correctly. It's pretty much always my goal to cover as many of these topics as possible without making the test too long.

ITEMS I MIGHT ASK YOU PROVE (Besides these I will not ask any other proof I did in lecture, however there will be a conceptual question which I have not asked before worth 4pts. This proof will be worth 10pts I will choose one of these items):

1. Show that $\vec{x} = e^{\lambda t}\vec{u}$ is a nonzero solution to $\vec{x}' = A\vec{x}$ if we require that λ and $\vec{u}$ are constant with $det(A - \lambda I) = 0$ and $(A - \lambda I)\vec{u} = 0$. You will need to use a theorem from linear algebra which states that $B\vec{u} = 0$ has more than one solution only if $det(B) = 0$.

2. Show that if $\vec{w} = Re(\vec{w}) + iIm(\vec{w})$ is a solution to $\vec{w}' = A\vec{w}$ then both $Re(\vec{w})$ and $Im(\vec{w})$ are also solutions.

3. Show that if $\lambda = \alpha + i\beta$ and $\vec{u} = \vec{a} + i\vec{b}$ then

$$Re(e^{\lambda t}\vec{u}) = e^{\alpha t}\cos(\beta t)\vec{a} - e^{\alpha t}\sin(\beta t)\vec{b} \qquad and \qquad Im(e^{\lambda t}\vec{u}) = e^{\alpha t}\sin(\beta t)\vec{a} + e^{\alpha t}\cos(\beta t)\vec{b}.$$

   Notice that we then have found how to extract two real solutions from the complex solution. I should mention that I assume here that $\alpha, \beta, \vec{a}, \vec{b}$ are all real, they have no $i = \sqrt{-1}$.

4. Show that if X is a fundamental matrix for $\vec{x}' = A\vec{x}$ then $\vec{x_p} = X \int X^{-1}\vec{f}dt$ solves $\vec{x_p} = X \int X^{-1}\vec{f}dt$.

5. Show that the matrix exponential is a fundamental matrix. That is show that e^{At} is invertible and it is a solution matrix for $\vec{x}' = A\vec{x}$.