E1

$x'(t) - 2y(t) = 4t$	$x(0) = 4$
$y'(t) + 2y(t) - 4x(t) = -4t-2$	$y(0) = -5$

Use $\mathcal{L}\{x\} = X$ and $\mathcal{L}\{y\} = Y$ to denote Laplace transforms of x and y. Transform the given system

$$sX - 4 - 2Y = 4/s^2 \longleftarrow (*)$$
$$sY + 5 + 2Y - 4X = -4/s^2 - 2/s$$

That is,

$$sX - 2Y = 4 + 4/s^2$$
$$-4X + (s+2)Y = -5 - 4/s^2 - 2/s$$

Multiply the top eq$^{\underline{n}}$ by $(s+2)$ and the bottom eq$^{\underline{n}}$ by 2 to get

$$+\begin{pmatrix} s(s+2)X - \cancel{2(s+2)Y} = (4 + 4/s^2)(s+2) \\ -8X + \cancel{2(s+2)Y} = -10 - 8/s^2 - 4/s \end{pmatrix}$$

$$(s(s+2) - 8)X = 4\left(\frac{s^2+1}{s^2}\right)(s+2) - 10 - \frac{8}{s^2} - \frac{4}{s}$$

$$X = \frac{1}{s^2+2s-8}\left[4(s+2)\frac{(s^2+1)}{s^2} - 10 - \frac{8}{s^2} - \frac{4}{s}\right]$$

$$= \frac{1}{(s-2)(s+4)}\left[\frac{4(s+2)(s^2+1) - 10s^2 - 8 - 4s}{s^2}\right]$$

$$= \frac{1}{(s-2)(s+4)}\left[\frac{4s^3 + \cancel{4s} + 8s^2 + \cancel{8} - 10s^2 - \cancel{8} - \cancel{4s}}{s^2}\right]$$

$$= \frac{4s-2}{(s-2)(s+4)} = \frac{3}{s+4} + \frac{1}{s-2} \quad \text{(using partial fractions)}$$

Thus $\boxed{x(t) = 3e^{-4t} + e^{2t}}$. Let's use the eq$^{\underline{n}}$ $(*)$ to find Y so we can then use $\mathcal{L}^{-1}$ to compute y.

$$Y = \tfrac{1}{2}sX - 2 - 2/s^2$$

$$= \frac{s}{2}\left(\frac{3}{s+4} + \frac{1}{s-2}\right) - 2 - \frac{2}{s^2}$$

$$= \frac{3}{2}\frac{s}{s+4} + \frac{1}{2}\frac{s}{s-2} - 2 - \frac{2}{s^2}$$

E1 Continued,

$$\mathcal{Y} = \frac{3}{2}\left(\frac{s+4-4}{s+4}\right) + \frac{1}{2}\left(\frac{s-2+2}{s-2}\right) - 2 - \frac{2}{s^2}$$

$$= \cancel{\frac{3}{2}} - \frac{6}{s+4} + \cancel{\frac{1}{2}} + \frac{2}{s-2} - \cancel{2} - \frac{2}{s^2}$$

$$= \frac{-6}{s+4} + \frac{2}{s-2} - \frac{2}{s^2}$$

Thus it is clear that

$$y(t) = \mathcal{L}^{-1}\{\mathcal{Y}\}(t) = \boxed{-6e^{-4t} + 2e^{2t} - 2t}$$

- the text found the same answer a different way.

Summary: we can solve systems of ODEs by transforming the eq^n's to the s-domain, doing algebra, then taking inverse transform back to the t-domain. We'll find a systematic method for solving linear systems of ODEs with the remainder of the course. The method we used here is not as clean as the methods developed in chpt. 9 in my opinion.