Math 341
Exam 1

Name:______________________

July 17, 2006

This exam contains 8 problems. Do all of your work in this booklet.
DO NOT SHOW YOUR WORK IN PROBLEMS 1–3 (no partial credit).

Problem 1

(3pts) Decide whether the method of undetermined coefficients together with superposition can be applied to find a particular solution of the given equations. **Do not solve the equations.**

Equation	Yes	No
$2y'' - y' + 5ty = te^t \cos t - e^{2t}$	◯	◯
$y'' + 3y' - 4y = t/\sin t - t^2 e^t$	◯	◯
$y'' + y' + 2y = 2te^t - t^2/e^t$	◯	◯

Problem 2

(6pts) Consider the the following statements, answer true or false by marking appropriate circle.

Equation	True	False
$\cos(x)$ and $\sin(x)$ are linearly dependent	◯	◯
e^x and xe^x are linearly independent	◯	◯
$x^2y'' + xy' + y = \sin^2(x)$ is a linear O.D.E.	◯	◯
$A = D^2 + 4$ anihilates $\cos(x)$	◯	◯
If $A[f] = 0$ then f is a sol^n to $AL[y] = 0$ (A, L are linear diff. operators)	◯	◯
$[2x + y\cos(xy)]dx + [x\cos(xy) - 2y]dy = 0$ is an $exact\ eq^n$	◯	◯

Problem 3

(6pts) Use the method of undetermined coefficients to find the **form of a particular solution** for the following differential equations. **Do not solve the equations.**

(a) $y'' - 2y' + 5y = te^{2t} \sin t$

Answer:

(b) $(\boldsymbol{D}^2 + 1)(\boldsymbol{D} + 3)^2 y = x^2 e^{-3x} + cos(x)$ where $\boldsymbol{D} = d/dx$ and $\boldsymbol{D}^n$ indicates the n-fold composition of the operator $\boldsymbol{D}$.

Answer:

In problems 4–8 partial credit will be given. SHOW YOUR WORK.

Problem 4

(15pts) Find the general solution of the equation below assuming $x > 0$,

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1$$

Problem 5

(15 pts) Assume that a particle of mass m is undergoing horizontal motion. Further assume it is subject to a frictional force which is proportional to the cube of it's velocity. This leads to the following differential equation for the particle's position x,

$$mx'' = kv^3$$

Find the position, velocity and acceleration of the particle at time t assuming the intial conditions $x(0) = 0$ and $x'(0) = 10$.

Problem 6

(15pts) Solve the initial value problem $y'' + 2y' - 8y = 0$, $y(0) = 3$, $y'(0) = 0$.

Problem 7

(20pts) Use the **method of variation of parameters** to find the general solution of the equation

$$y'' + 4y = \tan(2x).$$

Problem 8

(20pts) Use the method of undetermined coefficients and the superposition principle to find the general solution to

$$y'' + 4y = x^3 + x^2 + 2 + \tan(2x)$$

Notice that the preceding problem is **very** relevant.