Ma 430, Problems Due Thursday 8/31/06

Problem 1 Show that $(A \times B) \cdot B = 0$ using the repeated index notation. A and B are vectors in $\mathbb{R}^3$.

Problem 2 Verify the identity $\epsilon_{ijk}\,\epsilon_{mjk} = 2\delta_{im}$ in the particular cases below

(a.) $i = 1$, $m = 1$
(b.) $i = 1$, $m = 2$
(c.) $m = 1$, $i = 3$

Problem 3 Verify the identity $\epsilon_{ijk}\,\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{jl}\delta_{im}$ in the particular cases below

(a.) $i = 1$, $j = 2$, $l = 1$, $m = 1$
(b.) $i = 1$, $j = 2$, $l = 1$, $m = 2$
(c.) $i = 1$, $j = 2$, $l = 2$, $m = 3$

Problem 4 Use the identities in Problems 2 & 3 plus the antisymmetry of ϵ_{ijk} to find nice formulas for the following in terms of δ_{mk}, or a constant.

(a.) $\epsilon_{kij}\,\epsilon_{mjk}$
(b.) $\epsilon_{abd}\,\epsilon_{def}$
(c.) $\epsilon_{ijk}\,\epsilon_{ijk}$
(d.) $\epsilon_{kij}\,\epsilon_{lmk}$

Problem 5 Prove parts (viii) and (ix) of Proposition 3.3.1

Problem 6 Prove proposition 3.3.2.

Problem 7 Work Energy Theorem. Let $K_i = \frac{1}{2} m v_i^2$ and let $F = m\frac{dv}{dt}$ where m is a constant. Show that

$$K_f - K_i = \int_{x(t_i)}^{x(t_f)} F(x)\,dx$$

(Hint: $v = dx/dt$, $\frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx}$)