# Ma 430 : Hodge Duality and Exterior Algebra

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$\boxed{\text{PROBLEM 37}}$ I mentioned in lecture $\dim(\Lambda^p(V)) = \dfrac{n!}{(n-p)!\,p!}$ where $\dim(V) = n$. Verify that when $V = \mathbb{R}^3$ we have $\dim(\Lambda^1(V)) = \dim(\Lambda^2(V)) = \dim(V)$. Study what happens when $n \neq 3$. Why is the case $n = 3$ so special? If you can find a general argument then great, otherwise provide enough examples to convince yourself.

$\boxed{\text{PROBLEM 38}}$ Hodge duality provides the following diagram

$$\Lambda^p(V) \xrightarrow{\;*\;} \Lambda^{n-p}(V) \longrightarrow \Lambda^p(V)$$

something like $** = (-1)^{np-s}$

where $V$ is a $n$-dim'l vector space with a metric $g$. Note

$$**\alpha = (-1)^{BOB}\alpha$$

where this is sometimes abbreviated $(*)^2 = (-1)^{BOB}$. Here BOB will depend on several influences. Let us suppose $\alpha \in \Lambda^p(V)$

i.) find BOB in the case $V = \mathbb{R}^3$ and $g_{ij} = \delta_{ij}$

ii.) find BOB in the case $V = \mathbb{R}^4$ and $g_{\mu\nu} = \eta_{\mu\nu}$

iii) find BOB in the case $V = \mathbb{R}^4$ and $g_{\mu\nu} = -\eta_{\mu\nu}$.

$\boxed{\text{PROBLEM 39}}$ Complete the discussion on pg. 113. That is show,

$$\boxed{\begin{aligned} *dy &= dz \wedge dx \\ *dz &= dx \wedge dy \end{aligned}} \quad \& \quad \boxed{\begin{aligned} *(dz \wedge dx) &= dx \\ *(dx \wedge dy) &= dz \end{aligned}}$$

$\boxed{\text{PROBLEM } \cancel{35}\ 40}$ Verify my claim that for vectors $A, B \in \mathbb{R}^3$

$$\boxed{\omega_A \wedge \omega_B = \Phi_{A \times B}}$$

See pg. 114 for definitions of $\omega : \mathbb{R}^3 \longrightarrow \Lambda^1(\mathbb{R}^3)$ the "work form" mapping $\Phi : \mathbb{R}^3 \longrightarrow \Lambda^2(\mathbb{R}^3)$ the "flux form" mapping.

$\boxed{\text{PROBLEM } \cancel{36}\ 41}$ Show that $\dim(\Lambda(V)) = 2^{\dim(V)}$.