Ma 430 : Hodge Duality in Minkowski Space (TAKE-HOME EXAM II)

PROBLEM 42 Work out the remaining Hodge Dualities in Minkowski Space,

$$*dy = -dz \wedge dx \wedge dt$$
$$*dz = -dx \wedge dy \wedge dt$$
$$*(dz \wedge dt) = dx \wedge dy$$
$$*(dy \wedge dt) = dz \wedge dx$$
$$*(dx \wedge dy) = -dz \wedge dt$$
$$*(dy \wedge dz) = -dx \wedge dt$$
$$*(dz \wedge dx) = -dy \wedge dt$$

PROBLEM 43 On pg. 98 in eq$^{\underline{n}}$ 9.15 we defined the field tensor at a point in Minkowski space. Verify that the following expressions also yield the same tensor

(a.) $F = \omega_{\vec{E}} \wedge dt + \Phi_{\vec{B}}$

(b.) $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

For part (b.) I remind you that $(A_\mu) = (-V, \vec{A})$ where V is the scalar potential and $\vec{A}$ is the vector potential, they relate to the $\vec{E}, \vec{B}$ fields as follows,

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \quad , \quad \vec{B} = \nabla \times \vec{A}$$

PROBLEM 44 Show that $d\phi = 0$ where $\phi = \frac{1}{x^2+y^2}(x\,dy - y\,dx)$. (Eq$^{\underline{n}}$ 11.32)

PROBLEM 45 Show that the field tensor is a closed form. Is the field tensor an exact form? You may assume the result of Griffith's text, given a particular charge & current configuration $\exists$ a globally defined scalar & vector potential.

PROBLEM 46 Show that $A = A_\mu dx^\mu$ and $A' = A + d\lambda$ yield the same field tensor.

PROBLEM 47 Before we discussed the Coulomb Gauge $\nabla \cdot A = 0$ and the Lorentz gauge $\partial_\mu A^\mu = 0$. Which of these gauge choices is preserved under a Lorentz transformation?

PROBLEM 48 Show that if a charge q is at rest with $\vec{B} = B\hat{z}$ in S then it is in constant velocity motion in S' where S' is x-boosted S.