Ma 430, Problems Due 9/7/06

Problem 8 If someone tells you that the magnetic field in some region is $\vec{B} = \alpha x \hat{\imath}$, where $\alpha > 0$ and $\hat{\imath}$ is unit-vector in x-direction, should you believe him? Yes, no and why (explain).

Problem 9 In the notes we claimed that there exist potential functions V and $\vec{A}$ such that

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \nabla \times \vec{A}$$

show that the functions $\vec{A}$ and V are not unique, that is show that $\vec{A}' = A + \nabla \lambda$ and $V' = V - \frac{\partial \lambda}{\partial t}$ also give the same $\vec{E}$ and $\vec{B}$.

Problem 10 Derive the following partial differential eq^{n}'s for the potentials assuming Maxwell's Eq^{n}'s hold. That is translate Maxwell's Eq^{n}'s into the potential formulation,

$\nabla^2 V = -\rho/\epsilon_0$ [redacted] $\frac{\partial}{\partial t}(\nabla \cdot \vec{A})$

$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$ [redacted] $\mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} - \mu_0 \epsilon_0 \nabla\left(\frac{\partial V}{\partial t}\right)$

Problem 11 Find nice eq^{n}'s for $\vec{A}$ and V for the following gauge conditions". Use **Problem 10** as a starting point.

(a.) Coulomb Gauge : $\nabla \cdot \vec{A} = 0$.

(b.) Lorentz Gauge : $\nabla \cdot \vec{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

Useful Facts :- $\nabla^2 = \nabla \cdot \nabla$, $\nabla \frac{\partial}{\partial t} = \frac{\partial}{\partial t} \nabla$ for all of 9, 10, 11 perhaps.