

No graphing calculators or electronic communication of any kind. If you need extra paper please ask. Credit will be awarded for correct content and clarity of presentation. This test has 100 points. Try to at least attempt each part. Please clearly box your answer in each question.

1. [10pts] Multiply the polynomials and put your answer in standard form.

$$(x^2 + 3x - 2)(x^2 + 4x + 1) = x^4 + 4x^3 + x^2 + 3x^3 + 12x^2 + 3x - 2x^2 - 8x - 2$$

$$= \boxed{x^4 + 7x^3 + 11x^2 - 5x - 2}$$

2. [20pts] Completely factor (over \mathbb{R}) the following polynomials:

(a.) $x^2 - 10x + 9 = \boxed{(x-9)(x-1)}$

(b.) $x^3 + x = \boxed{x(x^2 + 1)}$

(c.) $x^3 - 9x = x(x^2 - 9) = \boxed{x(x+3)(x-3)}$

3. [30pts] Solve the following quadratic equations, the solutions may be real and distinct, real and repeated or possibly complex. Find the solutions for each equation below.

(a.) $x^2 - 10x + 9 = 0 \Rightarrow (x-9)(x-1) = 0 \therefore \boxed{x=9 \text{ or } x=1}$

(b.) $x^2 = 4 \Rightarrow x = \pm\sqrt{4} \Rightarrow \boxed{x = \pm 2}$

(c.) $x^2 + 6x + 10 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{2} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm 2i}{2}$$

$$\Rightarrow \boxed{x = -3 \pm i}$$

4. [10pts] Solve $\sqrt{x} + \sqrt{x-20} = 10$ for x .

$$(\sqrt{x-20})^2 = (10 - \sqrt{x})^2$$

$$x - 20 = 100 - 20\sqrt{x} - x$$

$$20\sqrt{x} = 120$$

$$\sqrt{x} = 6$$

$$\boxed{x = 36}$$

5. [10pts] Find the equation for the line passing through (1,0) and (2,3). Please leave your answer in slope-intercept form.

$$y = mx + b$$

$$(1,0): 0 = m(1) + b \Rightarrow \underline{b = -m}$$

$$(2,3): 3 = 2m + b \Rightarrow 3 = 2m - m = m$$

$$\Rightarrow m = 3 \text{ and } b = -3$$

$$\therefore \boxed{y = 3x - 3}$$

6. [10pts] Solve the following inequality. Justify your answer either with a sign chart as I used in lecture, or with a table as shown in the textbook. Remember to neatly box your answer.

$$\frac{x+12}{x+2} \leq 3 \Leftrightarrow \frac{x+12}{x+2} - 3 \leq 0$$

$$\Leftrightarrow \frac{x+12-3(x+2)}{x+2} \leq 0$$

$$\Leftrightarrow \frac{-2x+6}{x+2} \leq 0$$

critical #'s $x = -2$ and $x = 3$ ($-2x+6=0 \Rightarrow x=3$)

Thus the sign chart below is appropriate,



Thus we find (note $x = -2$ excluded because of division by zero whereas $x = 3$ is included since it's a zero of the given expression. Thus,

$$\boxed{x < -2 \text{ or } x \geq 3}$$

7. [10pts] What explicit numerical values for A, B that make the equality below true?

$$\frac{(x\sqrt{xy})^3}{y^{-2}x^{\frac{3}{2}}} = x^A y^B$$

$$x\sqrt{xy} = x\sqrt{x}\sqrt{y} = x^1 x^{\frac{1}{2}} y^{\frac{1}{2}} = x^{\frac{3}{2}} y^{\frac{1}{2}}$$

$$(x\sqrt{xy})^3 = (x^{\frac{3}{2}} y^{\frac{1}{2}})^3 = x^{\frac{9}{2}} y^{\frac{3}{2}}$$

$$\begin{aligned} \frac{(x\sqrt{xy})^3}{y^{-2}x^{\frac{3}{2}}} &= \frac{x^{\frac{9}{2}} y^{\frac{3}{2}}}{y^{-2}x^{\frac{3}{2}}} = x^{\frac{9}{2}-\frac{3}{2}} y^{\frac{3}{2}-(-2)} \\ &= x^3 y^{\frac{7}{2}} = x^A y^B \end{aligned}$$

$$\Rightarrow \boxed{A = 3 \text{ and } B = \frac{7}{2}}$$