Math 121, Test 2, Section # : Name: .

No graphing calculators or electronic communication of any kind. If you need extra paper please ask. Credit will be awarded for correct content and clarity of presentation. This test has 100 points. Try to at least attempt each part. Please clearly **box** your answer in each question.

1.[3pts] [section 2.2] Find the domain for the function.

$f(x) = \sqrt{x-4}$

need $x - 4 \geq 0 \therefore \underline{x \geq 4}$ or $\boxed{\text{dom}(f) = [4, \infty)}$

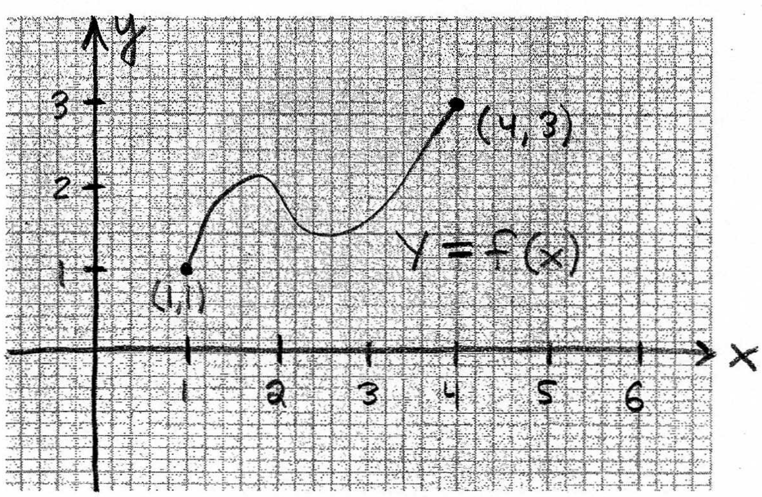
2.[2pts] [section 2.2] Suppose that $f(1) = 2$ and $f(1) = 3$ can f be a function? Why or why not?

No. A function has only one output for each input. This "function" would fail the vertical line tes

3.[5pts] [section 2.2] Find $(f \circ g)(2)$ given that $g(x) = x^2$ and $f(x) = \begin{cases} x+3 & \text{if } x \geq 3 \\ 0 & \text{if } x < 3 \end{cases}$

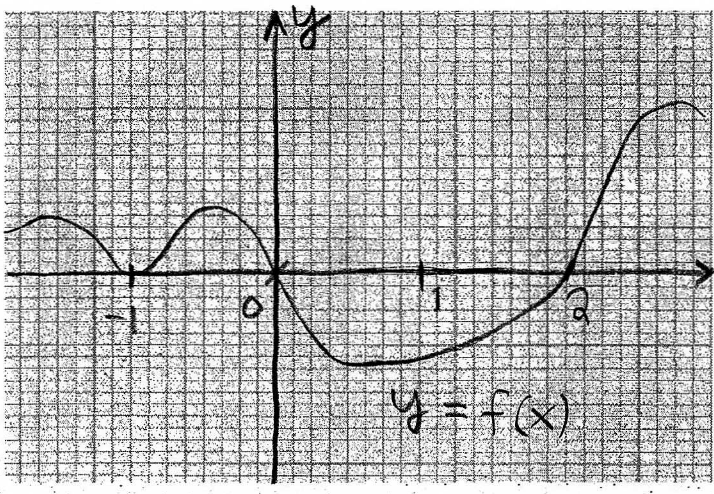
$$(f \circ g)(2) = f(g(2)) = f(4) = 4 + 3 = \boxed{7}$$

4.[5pts] [section 2.3] Find the domain and range of the graphed function.



$\boxed{\begin{array}{l} \text{dom}(f) = [1, 4] \\ \text{range}(f) = [1, 3] \end{array}}$

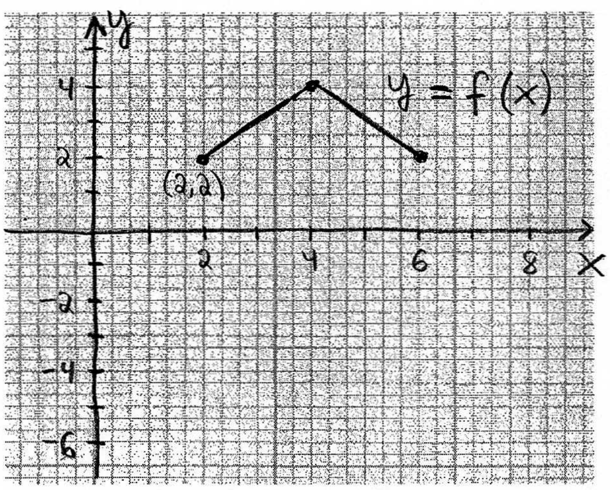
5.[5pts] [section 2.3] Find the zeros of the function $f(x)$ graphed below.

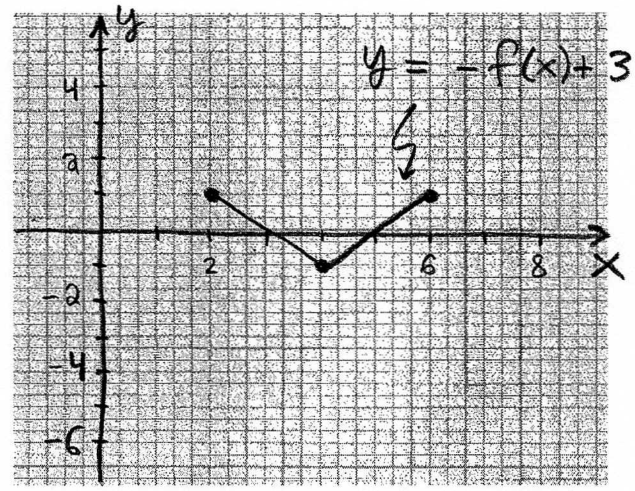


Zeros are at

$x = -1, 0, 2$

6.[5pts][section 2.5] Given the graph of $y = f(x)$ construct the graph of $y = -f(x) + 3$ in the blank xy-plane provided below.





$f(2) = 2 \Rightarrow -f(2) + 3 = -2 + 3 = 1$ etc...

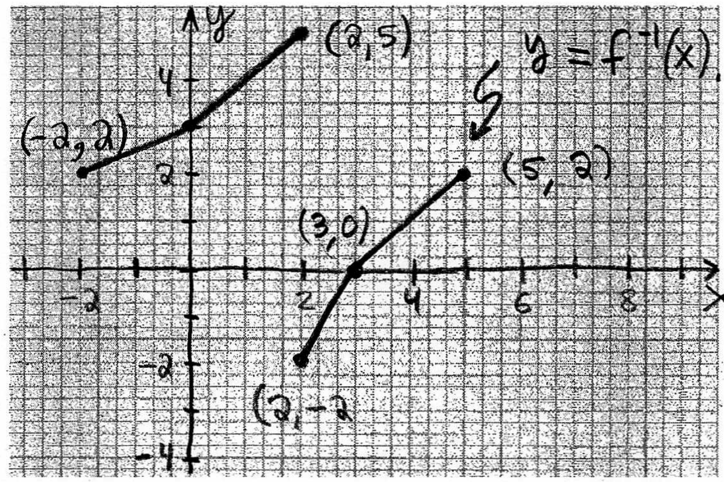
[15pts]
7. [[redacted]][section 2.6] Given that $f(x) = x^2$ and $g(x) = \sqrt{x-4}$ find the formulas for

(a.) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-4}) = (\sqrt{x-4})^2 = \boxed{x-4}$

(b.) $(f+g)(x) = f(x) + g(x) = \boxed{x^2 + \sqrt{x-4}}$

(c.) $(fg)(x) = f(x)\,g(x) = \boxed{x^2\sqrt{x-4}}$

8.[5pts][section 2.7] Graph the inverse function of the function graphed below.



9.[5pts][section 2.7] If $f(x) = 3 - 4x$ then find the formula for the inverse function $f^{-1}(x)$.

$$y = 3 - 4x$$

$$x = 3 - 4y \Rightarrow 4y = 3 - x$$

$$\Rightarrow y = \frac{1}{4}(3 - x)$$

$$\therefore \boxed{f^{-1}(x) = \frac{1}{4}(3 - x)}$$

10.[10pts] [section 3.1] Write the equation of a quadratic function whose graph has vertex $(-2, -2)$ and passes through the point $(-1, 0)$. Please put the answer in standard form.

A parabola with vertex (h, k) satisfies the equation $y = a(x-h)^2 + k$ thus $y = a(x+2)^2 - 2$. What is $a = ?$

Use the other point $(-1, 0)$ that yields

$$0 = a(-1+2)^2 - 2$$

$$\Rightarrow 0 = a - 2 \Rightarrow \underline{a = 2}$$

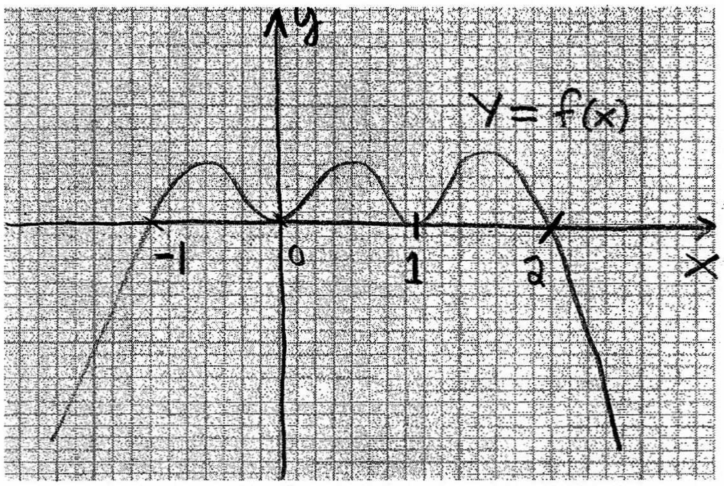
$$\boxed{\begin{aligned} y &= 2(x+2)^2 - 2 \\ &= 2x^2 + 8x + 6 = y \end{aligned}}$$

12.[5pts] [section 3.2] Write a formula for a polynomial function $f(x)$ which is of degree 3 and has zeros at $x = 1, 2, 3$.

$$\boxed{f(x) = (x-1)(x-2)(x-3)}$$

(many other correct answers.)

13. [10pts][section 3.2] Given the graph below circle each formula which could give such a graph.



(a.) $f(x) = -(x+1)x^2(x-1)^2(x-2)$

(b.) $f(x) = (x+1)x^2(x-1)^2(x-2)$

(c.) $f(x) = -(x+1)^3x^4(x-1)^4(x-2)$

(d.) $f(x) = x^2(x-1)$

(e.) $f(x) = \dfrac{42}{(x+1)x^2(x-1)(x-2)}$

(Notice (b) goes up not down, wrong sign)

14.[5pts][section 3.3] Does the function $f(x) = 33x^{13} - 20x^2 - 11x - 2$ have $(x-1)$ as a factor? Justify your answer either by a Theorem or by an explicit algebraic calculation.

Note $f(1) = 33 - 20 - 11 - 2 = 33 - 33 = 0$

$\therefore$ by factor Th$^{\underline{m}}$ $f(x) = (x-1)q(x)$. YES it does have $x-1$ as factor

15.[5pts][section 3.4]Completely factor $f(x) = x^3 + 11x^2 - x - 11$. (*Hint: notice that* $f(1) = 0$.)

If $f(1) = 0$ then $f(x) = (x-1)q(x)$. We find $q(x)$ by long division.

$$\begin{array}{r|l} & x^2 + 12x + 11 \\ \hline x-1 & x^3 + 11x^2 - x - 11 \\ & \underline{x^3 - x^2} \\ & \quad 12x^2 - x - 11 \\ & \quad \underline{12x^2 - 12x} \\ & \qquad 11x - 11 \\ & \qquad \underline{11x - 11} \\ & \qquad\quad 0 \end{array}$$

$\therefore f(x) = (x-1)(x^2 + 12x + 11)$

$= \boxed{(x-1)(x+1)(x+11)} = f(x)$

16.[5pts][section 3.4] Find a polynomial function $p(x)$ with real coefficients that has zeros $x = i, 2, -3$. Notice that $i = \sqrt{-1}$ so $i^2 = -1$.

Recall $x = i$ a zero $\Rightarrow$ $x = -i$ also a zero

Thus,

$$f(x) = (x+i)(x-i)(x-2)(x+3) \leftarrow$$ almost correct but has complex coefficient

$$= \boxed{(x^2+1)(x-2)(x+3) = f(x)}$$

17.[10pts][section 3.4] Completely factor $f(x) = x^4 + 5x^3 + 7x^2 + 5x + 6$ given that $f(x)$ has a zero at $x = i = \sqrt{-1}$.

Again if $f(i) = 0$ then $f(-i) = 0$ thus $(x+i)(x-i) = x^2+1$ is a factor of $f(x)$.

We need to find $g(x)$ such that $f(x) = (x^2+1)g(x)$

Long division

$$\begin{array}{r|l} & x^2 + 5x + 6 \\ \hline x^2+1 & x^4 + 5x^3 + 7x^2 + 5x + 6 \\ & x^4 + x^2 \\ \hline & 5x^3 + 6x^2 + 5x + 6 \\ & 5x^3 + 5x \\ \hline & 6x^2 + 6 \\ & 6x^2 + 6 \\ \hline & 0 \end{array}$$

Thus we find, $f(x) = (x^2+1)(x^2+5x+6)$

Factoring yields, $\boxed{f(x) = (x^2+1)(x+3)(x+2)}$