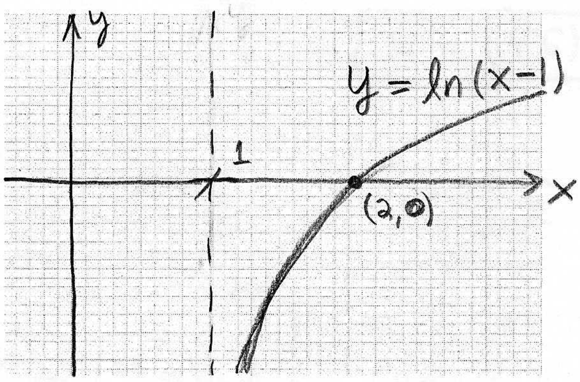
Math 121, Test 3, Section # : Name:

No graphing calculators or electronic communication of any kind. If you need extra paper please ask. Credit will be awarded for correct content and clarity of presentation. This test has 100 points. Try to at least attempt each part. Please clearly **box** your answer in each question.

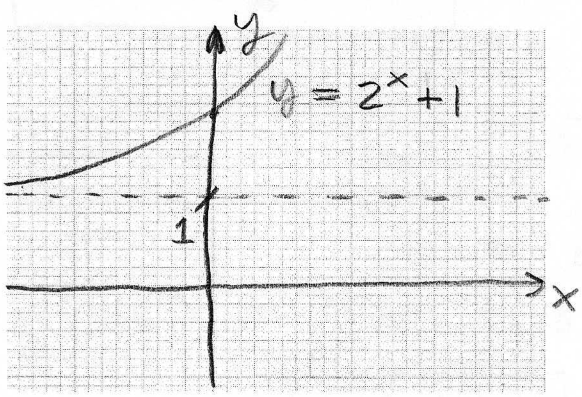
1.[5pts] Find the domain and range of $f(x) = \ln(x-1)$. Also, graph the function.



$\text{dom}(f) = (1, \infty)$

$\text{range}(f) = (-\infty, \infty) = \mathbb{R}$

2.[5pts] Find the domain and range of $f(x) = 2^x + 1$. Also, graph the function.



3.[10pts] Solve the equation:

$e^{x^2-6x} = e^{-5}$

$$\Rightarrow x^2 - 6x = -5$$

$$\Rightarrow x^2 - 6x + 5 = 0$$

$$\Rightarrow (x-1)(x-5) = 0$$

$$\therefore \boxed{x = 1} \ \& \ \boxed{x = 5}$$

4.[5pts] Solve the equation:

$3e^{x+2} = 81 \longrightarrow e^{x+2} = 27$

$\ln(e^{x+2}) = \ln(27)$

$x+2 = \ln(27)$

$\boxed{x = \ln(27) - 2}$

5.[5pts]Solve the equation:

$$\frac{3000}{2+e^{2x}} = 2$$

$3000 = 4 + 2e^{2x}$

$\frac{2996}{2} = e^{2x}$

$1498 = e^{2x}$

$\ln(1498) = \ln(e^{2x}) = 2x$

$\therefore \boxed{x = \frac{1}{2}\ln(1498)}$

6.[5pts]Solve the equation:

$\log_5\left(\sqrt{x-8}\right) = 2 \implies 5^{\log_5(\sqrt{x-8})} = 5^2$

$\sqrt{x-8} = 25$

$x - 8 = (25)^2 = 625$

$\therefore \boxed{x = 633}$

7. [5pts]Find values for A, B, C that make the equation below true.

$$\boxed{5\ln(x^2) + \frac{1}{7}\ln(x+6) - 3\ln(x^2+1) = \ln\left(\frac{x^A(x+6)^B}{(x^2+1)^C}\right)}$$

$\ln(x^{10}) + \ln((x+6)^{1/7}) + \ln\left(\frac{1}{(x^2+1)^3}\right) = \ln\left(\frac{x^{10}(x+6)^{1/7}}{(x^2+1)^3}\right)$

$\boxed{A = 10, \quad B = 1/7, \quad C = 3}$

8. [5pts]Find values for A, B, C that make the equation below true.

$$\boxed{\log\left(\frac{(x^2-9)^4}{(x+7)^{1.5}(x^2+1)^2}\right) = 4\log(x+A) + B\log(x-3) + C\log(x+7) - 2\log(x^2+1),}$$

$$\log\left(\frac{(x+3)^4(x-3)^4}{(x+7)^{1.5}(x^2+1)^2}\right) = 4\log(x+3) + 4\log(x-3) - 1.5\log(x+7) - 2\log_2(x^2+1)$$

$$\boxed{A=3,\ B=4,\ C=-1.5}$$

9.[10pts]Solve the following system of equations:

$$+\begin{pmatrix} x+2y=5 \\ -x+4y=7 \end{pmatrix}$$

$6y = 12 \quad \therefore \quad y = 2 \quad \Rightarrow \quad x = 5-2(2) = 5-4 = 1$

$\therefore \quad \boxed{x=1,\ y=2}$

10.[15pts]Solve the following system of equations:

$x + 2y + z = 4$: R1
$-3x + y - z = 0$: R2
$x - y - z = 0$: R3

R1+R3 : $2x + y = 4$ $\Longrightarrow$ $\begin{pmatrix} 4x + 2y = 8 \\ + \ 4x - 2y = 0 \end{pmatrix}$

R3−R2 : $4x - 2y = 0$

$8x = 8 \quad \therefore \ x = 1$

R1+R3 : $y = 4 - 2x = 4 - 2$
$\Rightarrow y = 2$

Then R3 yields
$z = x - y = 1 - 2 = -1$

Thus $\boxed{x=1,\ y=2,\ z=-1}$

11.[5pts]Given the information below, read off the solutions.

$$\begin{aligned} 3x - y + z &= 0 \\ x + 2y + z &= 4 \\ 2x - 2y - 2z &= 0 \end{aligned} \quad \text{has} \quad rref\begin{pmatrix} 3 & -1 & 1 & 0 \\ 1 & 2 & 1 & 4 \\ 2 & -2 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\boxed{\begin{aligned} x &= 1 \\ y &= 2 \\ z &= -1 \end{aligned}}$$

12.[5pts] Suppose that $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ has the multiplicative inverse $A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$. Solve the following system of equations given in matrix form below by multiplying by the inverse matrix given above.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \iff A\vec{x} = \vec{b} \Rightarrow A^{-1}A\vec{x} = A^{-1}\vec{b} \Rightarrow \underline{\vec{x} = A^{-1}\vec{b}}$$

thus $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$= \begin{bmatrix} 1 - 2 + 3 \\ 0 + 2 + 0 \\ 0 + 0 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix} \quad \therefore \quad \boxed{\begin{aligned} x &= 2 \\ y &= 2 \\ z &= -3 \end{aligned}}$$

Check: $x + y + z = 1$ $2 + 2 - 3 = 1$ ✓

$y = 2$ ✓

$-z = 3$ ✓

13. [16pts] Suppose that $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 8 \\ 0 & 2 \end{pmatrix}$. Calculate

a.) AB $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 8 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 12 \\ 15 & 32 \end{pmatrix}$.

b.) $A + 2B$ $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 5 & 8 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 10 & 16 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 11 & 18 \\ 3 & 8 \end{pmatrix}$.

c.) $\det(A)$ $= \det\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = 4 - 6 = -2 = \det A$

d.) A^{-1} $= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = \boxed{\frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}}$

(either is fine.)

14. [4pts] Calculate the following:

$\det \begin{pmatrix} 1 & 1 & 1 \\ -1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ $= 1 \det \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - 1 \det \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} + 1 \det \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$

$= 6 - (-3)$

$= \boxed{9.}$

<u>BONUS[5pts]:</u> Given that the message below was encoded using $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ find the hidden message in 25, 11, 4, 2, 5, -5.

$\longrightarrow A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$\frac{1}{2}[25, 11]\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2}[25-11, 25+11] = [7, 18]$

$\frac{1}{2}[4, 2]\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2}[4-2, 4+2] = [1, 3]$

$\frac{1}{2}[5, -5]\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2}[5-(-5), 5-5] = [5, 0]$

$\boxed{\text{GRACE}}$