Math 131 Homework Quiz VI: (Stewart Calculus Ed. 6) September 1, 2008.

Open notes open book, no group work. Show work where appropriate. (problem on back, turn over)

2.3#10a) What is wrong with the following equation? *(where does it fail?)*

$$\frac{x^2+x-6}{x-2} = x+3$$

***Solution:*** we must avoid $x = 2$ because of division by zero.

2.3#10b) In view of part (a), explain why the following equation is correct.

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} x+3$$

***Solution:*** the limit gets close to $x = 2$ but it does not actually evaluate at two, this is the beauty of the limit. In particular,

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x\to 2} x+3.$$

2.3#61) Is there a number $a$ such that the limit below exists? If so, find the value of $a$ and the value of the limit. $\lim_{x\to -2} \dfrac{3x^2+ax+a+3}{x^2+x-2}$

***Solution:*** Observe that we will have division by zero at the limit point. Clearly $x^2+x-2 = (x+2)(x-1)$ so if we can cancel the offending factor by a corresponding factor in the numerator then we will find the limit is finite. In particular we would like to see

$$3x^2+ax+a+3 = (x+2)(Ax+B)$$

I cannot see what $A, B$ ought to be without some more work. Some of you could see that $A = 3$ just eyeballing it. Lets continue, $3x^2+ax+a+3 = (x+2)(Ax+B) = Ax^2+Bx+2Ax+2B$

From which it follows that $A = 3,\ a = B+6,\ a+3 = 2B$. Now solve and substitute,

$$B = a-6 \implies a+3 = 2(a-6) \implies \boxed{a = 15}.$$

Thus, $3x^2+ax+a+3 = (x+2)(3x+15) = 3(x+2)(x+3)$. Use this algebra to compute,

$$\lim_{x\to -2} \frac{3x^2+ax+a+3}{x^2+x-2} = \lim_{x\to -2} \frac{3(x+2)(x+3)}{(x+2)(x-1)} = \lim_{x\to -2} \frac{3(x+3)}{x-1} = \boxed{-1}.$$